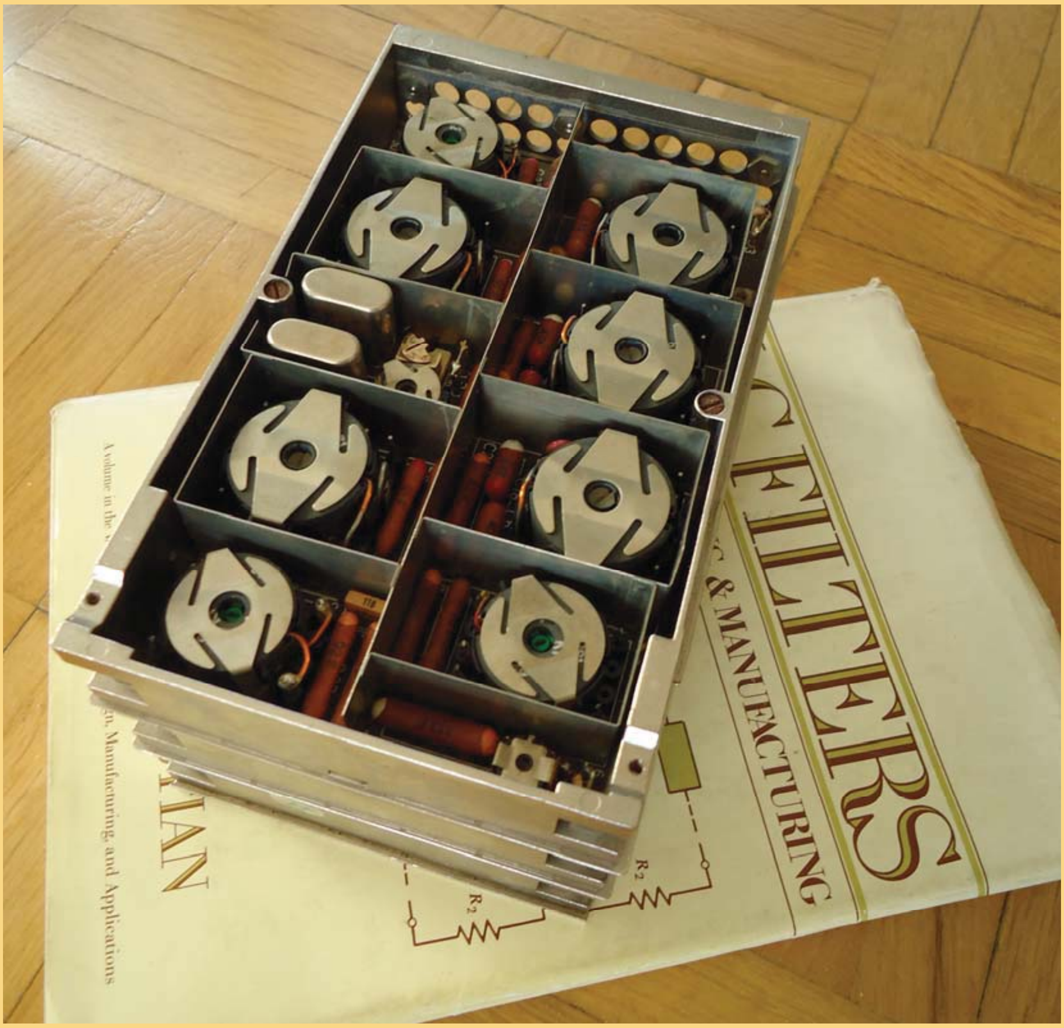
INTRODUCTION TO THE DESIGN OF TRANSMISSION NETWORKS

by

Erich Christian



1975
ITT Telecommunications
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About the author

Erich Christian developped his professional career as manager at ITT Telecommunications in Raleigh, North Carolina. He also served as Adjunct Professor at North Carolina State University. He earned his doctorate in network design at the University of Bochum (West Germany), and held a diploma in Physics from the Vienna Institute of Technology. He has been the author of two other books on network and filter design, a field he taught with brilliance.

INTRODUCTION TO THE DESIGN OF LOSSLESS TRANSMISSION NETWORKS

Erich Christian
ITT Telecommunications
and
North Carolina State University

Dedicated to

Die Wiener Techische Hochschule

and

N.C. State University

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ERRATUM

page 2 — line 17 "This chapter covers..." replace by "They cover.."

page 5 — after equ. (II. 6) : "conventional" replace by "convenient"

page 6 — after equ. (II. 9), line 4: ..., that they must either... replace by "..., that the roots must be either...."

page 6 — equ. (II. 11) :

$$\ldots\ldots\ldots \frac{F(s)\ F(-s)}{E(s)\ E(-s)}$$

page 11 — equation (unnumbered): $P_t = P_2 =$

$$E(s)\ E(-s) = \underbrace{\sqrt{1.2426}\ (s^2 + ps + q)}_{E(s)}\ \underbrace{\sqrt{1.2426}\ (s^2 - ps + q)}_{E(-s)}$$

page 20 — equ. (III. 8): $\ell_a \ell_b + \ell_a \ell_c + \ell_b \ell_c = 0$

page 26 — equ. (IV. 14 and 15): in all expressions, replace the letter "i" by the numeral "1".

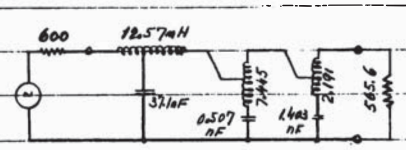
page 35 — Add to the text:

Since then, the program has also been made operational at several computer centers in the USA.

(Note: During recent months, errors were discovered in SYNTH2 in connection with the removal of pole-quadruplets. These have been corrected. As of this date, Nov. 1975, the latest version which is believed to be error-free is operational at TUCC (= Triangle University Computing Center, Research Triangle Park, Raleigh, N.C.). A tape copy of the program can be obtained from N.C. State University. Department of Electrical Engineering, Raleigh, N.C. 27607.

page 38 — Add the final network configuration:

```
     .      .
     .      .
     .      R      R( 1) =     600.00000000 OHMS
     .      .
     .      L      L( 2) =       3.22170650 MH
     .      .
     .      .      C( 3) =      37.11554500 NF
     ..C....L..    L( 3) =      -2.39660791 MH
     .      .      F( 3) =     -16.87499881 KHZ
     .      .
     .      L      L( 4) =       9.35785556 MH
     .      .
     .      L      L( 5) =      -1.48952119 MH
     .      .
     .      .      C( 6) =       0.50765162 NF
     ..C....L..    L( 6) =       5.28600448 MH
     .      .      F( 6) =      96.24998569 KHZ
     .      .
     .      L      L( 7) =       2.06891365 MH
     .      .
     .      L      L( 8) =      -0.31251522 MH
     .      .
     .      .      C( 9) =       1.40338074 NF
     ..C....L..    L( 9) =       1.81400730 MH
     .      .      F( 9) =      99.74999428 KHZ
     .      .
     .      L      L(10) =       0.37756103 MH
     .      .
     ....R.....    R(11) =     565.61244887 OHMS
```



Erratum (cont'.)

page 48	Move equation(V.14) over to the right such that the equal sign lines up with the vertical above
page 60	equ.(V.36): Enclose both expressions to the right of (A) in matrix brackets
page 75	Last paragraph, line 2: Therefore, if a characteristic function behaves in a certain way along the imaginary axis of the s-plane, it will display the same behaviour along the real axis of the s_1-plane
page 83	2^{nd} paragraph: For n=k, it is possible…
page 84	Equ. (V.84): $C_{ij} = C_{ji} = \ldots$
page 98	In the figure, indicate a normalized generator impedance"1"; also, in equ. (VI.28), replave V1 by V_1.
page 100	After the expression z11 = ….. , insert the sentence: z_{11} pertains to the left hand configuration , z_{22} to the right hand configuration.
page 102	In the normalized LC-circuit indicate the terminations as "1" on the left hand side, "0" on the right hand side
page 105	2^{nd} paragraph, line 6: "two-parts" – replace by "two-ports"
page 108	2^{nd} line from bottom: "Bruen"- replace by "Brune"
page 113	Line 6 : "They should be considered at least for some of the sections in an otherwise…" 2^{nd} paragraph, line 9: strike out the word "Consequently".
page 118	Line 7 : insert a comma before ω_2

I. Introduction

1. Background and scope

The design of electrical transmission networks began to play an increasingly important role ever since people tried to use well understood electrical phenomena to transmit messages, including the human voice. Historically first in this effort were transmission networks composed only of inductors and capacitors. The design methods for these transmission networks, known as "image parameter methods" were derived from the theory of transmission lines. Some of the notations of this theory, for instance the use of hyperbolic functions, have been carried over to modern design methods.

The early design methods began to disappear gradually as much more powerful design methods were published about the year 1940. However, because these new methods required a large number of calculations which had to be carried out with high precision, they did not make their breakthrough until computers came to their rescue. Now these computers are available and in wide-spread use and, therefore, the old theories are almost forgotten.

For a long period, the field of transmission networks was dominated almost entirely by circuits composed of capacitors, inductors and transformers only. Also included in this field can be transmission networks which contain mechanical resonators such as quartz crystals, because an equivalence can be established between such resonators and electrical circuits.

For a number of reasons, lossless transmission networks are now in competition with other devices which can accomplish the overall objective by different means. These are the devices known as "active filters" and "digital filters" which in some applications can outperform conventional LC devices. However, the design theory and methods for LC devices will maintain their importance for the following reasons:

(a) In a majority of applications, LC networks still offer the most economical and reliable solution. Modern components, for instance high quality ferrites and miniature capacitors, make this possible.

(b) Suitably designed LC networks can often be the basis for networks containing no coils or only a very few. These sometimes unsuitable or undesirable components can be replaced physically by devices such as quartz crystals or active-RC gyrators.

(c) The design concept for LC networks can also easily be modified towards the design of either active-RC filters or digital filters.

These considerations justify a closer study of the design methods for LC networks even if one contemplates transmission networks of a different type.

The synthesis procedure for transmission networks is carried out in two consecutive steps. The first step, called "the approximation” is a strictly mathematical task to determine suitable rational functions of a complex variable. These functions are related to the transfer performance of the transmission network. The name "approximation" implies that deviations from a desired performance must be expected and, therefore, realistic tolerance limits should be applied.

Because the transfer performance is related to certain rational functions, the mathematical task consists of selecting a suitable set of poles and zeroes for these functions, and a constant. This task is common to all types of networks regardless of the network type. With these functions the second step in the design can be carried out. It is commonly termed “the realization", implying the realization of suitable circuits. The pertinent procedure depends entirely on the desired network type.

The following sections contain a concise summary of the design based on synthesis methods. This chapter covers the approximation and the realization of conventional LC circuits. In order to avoid duplication with available publications in this field, reference will be made to some of the most commonly used text books and references. The theoretical expositions will be followed by rather simple demonstrative examples for which the calculations are easily carried out numerically with desk calculators or even with the slide rule. For more complicated examples, reference will be made to an existing computer program "SYNTH" which is available from the North Carolina State University, Department of Electrical Engineering, Raleigh, N. C. This program vas developed as an aid for a graduate course on the design of transmission networks. Its purpose was to teach students to make reasonable choices of pole and zero patterns to satisfy typical practical applications. Because the computer eliminates the need for rather cumbersome calculations, the student can spend more time on the creative aspects of the problems. The close agreement between calculated and the measured results never failed to convince the students of the value of synthesis methods. In combination with published parameter tables, the program can also become a powerful design tool for many applications.

For tutorial reasons, the derivations and the discussions are all carried out in the conventional s-plane. However, even with simpler networks the accumulated errors during calculations may make the results either highly inaccurate or even worthless.

Therefore, the critical calculations must be performed with sufficiently high precision. These precision requirements can be reduced considerably by a method known as "z-plane transformation". A brief outline of this method concludes the discussion of conventional LC filters.

Over many years and by many experts, the methods presented in this chapter have been found to be extremely useful for the design of transmission networks regardless of complexity. Because the results are highly reliable, the designer can with confidence predict the results, and have the assurance and the satisfaction that the transmission network will work as predicted. However, one should not accept just any results from the computer as the one and only answer. Unlike in network analysis, there may be several, hundreds or even thousands of correct solutions to a given problem. Network synthesis is one field where ingenuity, and also experience, has considerable weight. The proper selection from many possible solutions separates the excellent and good from the average and bad designer. The final criterion on the success of a solution will be its acceptance by the customer and its reliability in the system. To be successful in both requires more than a computer.

2. Normalization

Throughout this chapter, normalizations of frequencies, impedances and delays will be used in a slightly different manner than in most textbooks. For this reason, a concise summary on this subject is necessary.

Let $$Z = R + j\omega L + \frac{1}{j\omega C} \qquad \text{(I.1)}$$

be an arbitrary impedance. Furthermore, let

R_{ref} be an arbitrarily chosen reference resistance

f_{ref} be an arbitrarily chosen reference frequency

such that

$$f = \Omega f_{ref}\ , \ \omega = \Omega\,(2\pi f_{ref}) = \Omega\omega_{ref} \qquad \text{(I.2)}$$

where "Ω" is called the normalized frequency. By dividing both sides of equation (1) by R_{ref} it becomes

$$\frac{Z}{R_{ref}} = \frac{R}{R_{ref}} + j\Omega \frac{\omega_{ref} L}{R_{ref}} + \frac{1}{j\Omega \omega_{ref} C R_{ref}} \tag{I.3}$$

$$z = r + j\Omega \ell + \frac{1}{j\Omega c} \tag{I.4}$$

with

$$\ell = \frac{L}{L_{ref}}, L_{ref} = \frac{R_{ref}}{\omega_{ref}} \tag{I.5}$$

$$c = \frac{C}{C_{ref}}, C_{ref} = \frac{1}{R_{ref}\, \omega_{ref}}$$

In equation (I.5) the small case letters z, r, ℓ, c are normalized quantities of the impedance, the resistance, the inductance and the capacitance, respectively. All are dimensionless quantities. The quantities L_{ref} and C_{ref} are derived reference quantities and are related to the primary reference quantities R_{ref} and f_{ref} as shown in equation (I.5) As these, they are dimensioned factors of proportionality between the normalized and the actual circuit components. They also contain the factor "2π". In practice, this method of normalization and the inverse de-normalization is less confusing than others. A demonstration will be given after the first numerical examp1e in the next subsection.

II. The Power Transfer from a Generator into a Comp1ex Load

In this subsection, a detailed study will be made regarding the power transfer from a resistive generator into some complex impedance. According to the Helmholtz—Thevenin principle, the generator can be represented as an ideal voltage source in series with the internal impedance of the generator R. It is capable of delivering a maximum power P_{max}

$$P_{max} = |V_0|^2 / 4R = P_{ref} \tag{II.1}$$

into a matched load,(see Fig.II.1, part A.). As indicated in equation (II.1), this maximum power will be used as reference power P_{ref} in all considerations regarding power transfer.

1. Reflections

For arbitrary terminations, consider Fig. II.1, parts B and C. In part C the complex load is replaced by a matched load and an auxiliary voltage source V_r. Postulating that the currents are equal in both arrangements lead to

$$V_r = V_o \frac{R - Z}{R + Z} = \rho V_o \tag{II.2}$$

with
$$\rho = \frac{R - Z}{R + Z} \tag{II.3}$$

The quantity "ρ" is called: reflection coefficient. Part C of Fig. II.1 represents two resistive generators connected back to back. Because of the superposition principle of linear circuits, the two sources may be considered independently.

Power transfer from left to right:

$$P_{ref} = \frac{|V_o|^2}{4\ R_{ref}}$$

Power transfer from right to left:

$$P_r = \frac{|V_o \rho|^2}{4\ R_{ref}}$$

P_r is called the reflected power. The net power from left to right is the transmitted power P_t.

$$P_t = P_{ref} - P_r = P_{ref}(1 - |\rho|^2) \tag{II.4}$$

By definition,

Transducer loss A_t (dB) $= 10 \log_{10} \frac{P_{ref}}{P_t} = -10 \log_{10} (1 - |\rho|^2)$ (II.5)

Return loss A_r(dB) $= 10 \log_{10} \frac{P_{ref}}{P_r} = -20 \log_{10} |\rho|$ (II.6)

The transducer loss is a conventional measure to express the power into the terminating impedance and, eventually, through the transmission network. The return loss is a convenient measure to express the matching condition at the interface: generator - load. The better the load matches the generator, the smaller is the reflection coefficient and the higher the return loss. For instance, 26 dB return loss is about equivalent to a complex load which deviates not more than 10%, absolute, from the generator impedance.

2. The Transfer Polynomials

Let the generator impedance be chosen as R_{ref} ,then

$$\rho = \frac{R-Z}{R+Z} = \frac{1-\frac{Z}{R}}{1+\frac{Z}{R}} = \frac{1-z}{1+z} \qquad \text{(II.7)}$$

Furthermore, let the normalized impedance z

$$z = z(s) = \frac{n(s)}{d(s)} \qquad \text{(II.8)}$$

be a positive real function ([VA-1],pag.78), with n(s) and d(s) representing its numerator and denominator polynomials, respectively. Then, from equations (11.7) and (11.8)

$$\rho = \frac{1-z(s)}{1+z(s)} = \frac{d(s)-n(s)}{d(s)+n(s)} = \frac{F(s)}{E(s)} \qquad \text{(II.9)}$$

Because z(s) is positive real,

(a) The denominator polynomial d(s)+n(s)= E(s) must be a Hurwitz polynomial, i.e. a polynomial with roots only inside the left half of the s— plane.

(b) The numerator polynomial d(s)-n(s)= F(s) may or may not be a Hurwitz polynomial. One can only conclude that its coefficient must be real and no statement can be made regarding its root pattern except, of course, that they must either be real, or at the origin, or in conjugate complex pairs.

Expressed in terms of these polynomials

$$z = \frac{1-\rho}{1+\rho} = \frac{E(s)-F(s)}{E(s)+F(s)} \qquad \text{(II.10)}$$

Furthermore, for the magnitude of ρ along the imaginary axis, i.e. for $s = j\Omega$, one finds

$$\left|\rho(j\Omega)\right|^2 = \frac{F(s)*F^*(s)}{E(s)*E^*(s)}\Big|_{s=j\Omega} = \frac{F(s)*F(-s)}{E(s)*E(-s)}\Big|_{s=j\Omega} \qquad \text{(II.11)}$$

where the asterisks indicates the conjugate complex of the respective functions Also, for $s=j\Omega$, F*(s)= F(—s) and E*(s) = E(-s). Substituting equation (II. 11) in equation (11.4) yields

$$P_t(j\Omega) = P_{ref}\left[1-\frac{F(s)*F(-s)}{E(s)*E(-s)}\right]\Big|_{s=j\Omega} = P_{ref}\,\frac{E(s)E(-s)-F(s)F(-s)}{E(s)E(-s)}\Big|_{s=j\Omega}$$

$$= P_{ref} \left(\frac{E(s)E(-s) - F(s)F(-s)}{E(s)E(-s)} \right) \Bigg|_{s=j\Omega}$$

$$= P_{ref} \left(\frac{P(s)P(-s)}{E(s)E(-s)} \right) \Bigg|_{s=j\Omega} \qquad (II.12)$$

with $$P(s)\ P(-s) = E(s)\ E(-s) - F(s)\ F(-s) \qquad (II.13)$$

Substituting the expression P(s) P(-s) for the right side of equation (II.13) is justified only if the even polynomial on the right side can be separated in this manner. This implies that its root pattern has quadrantal symmetry (which it does) satisfied and also that roots on the imaginary axis are of even multiplicity. However, because P_t can not change sign for actual frequencies, i.e. for argument values s on the imaginary axis, such roots must necessarily have even multiplicity. The three polynomials E(s), F(s) and P(s) will be called "transfer polynomials". Significant for them is

(a) F(s) is a polynomial with real coefficients. According to equation (II.9), its roots are those, in general complex, frequencies for which no power is reflected, i.e. relection zeroes. At those, the transducer loss is zero.

(b) P(s) is also a polynomial with real coefficients. In addition to this it must also be either an even or an odd polynomial which will be proven later. According to equation (II.12), its roots are those frequencies for which no power is transmitted, i.e. the transmission zeroes which will later also be called attenuation poles.

(c) E(s) is a Hurwitz polynomial. Is roots are the natural modes of the transmission network which will also become apparent later.

In order to be compatible transfer polynomials must satisfy equation (II.13) which is conventionally written in the form

$$E(s)E(-s) = F(s)F(-s) + P(s)P(-s) \qquad (II.14)$$

The compatibility of transfer polynomials with a high precision is very important for any subsequent realization of LC networks. To determine the third polynomial when two are specified is usually responsible for an essential portion of the calculations.

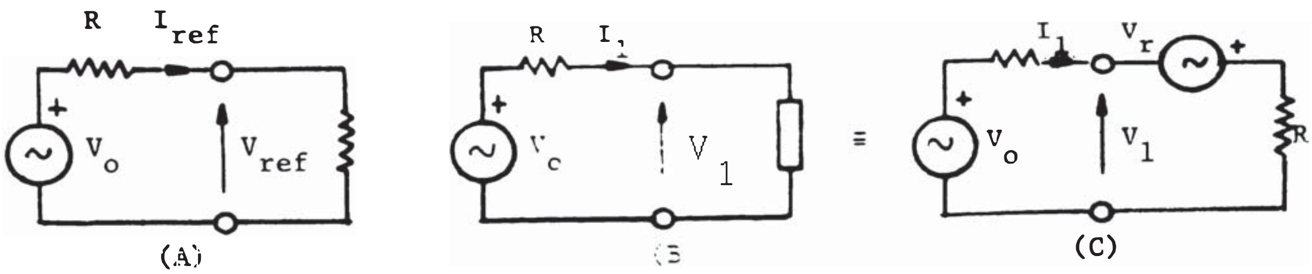


Figure II.1 Feldtkeller's Reflection Principle

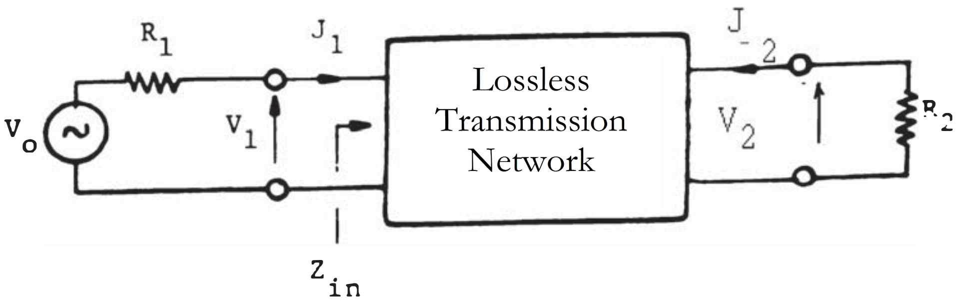


Figure III.1 A double terminated Lossless Transmission Network

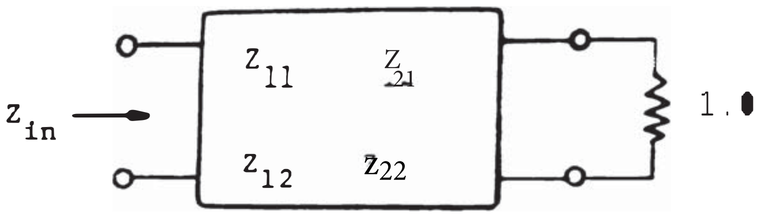


Figure III. 2 The normalized Input Z of the Transmission Network

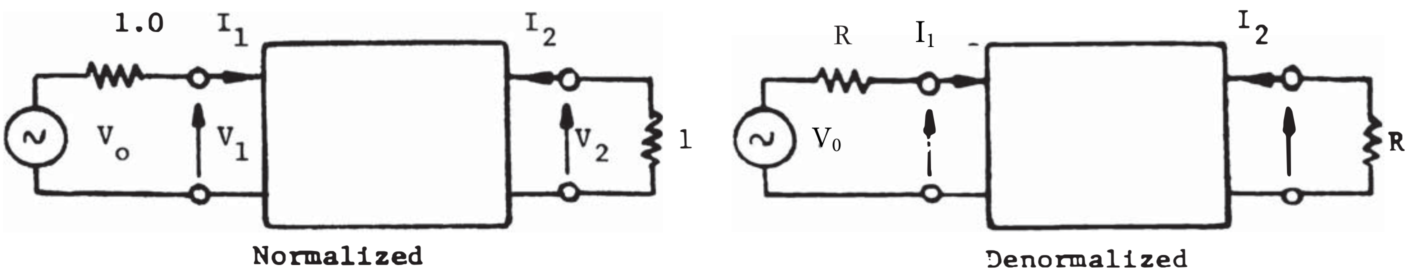


Figure III.4 : Transmission Network with Equal Terminations

3. Transfer function and characteristic function

By means of equations (II.5) and (II.12), the transducer loss can also be expressed in the form

$$A_t(dB) = 10 \log_{10} \frac{P_{ref}}{P_t} = 10 \log_{10} \left. \frac{E(s)E(-s)}{P(s)P(-s)} \right|_{s=j\Omega} \quad (II.15)$$

$$A_t(dB) = 10 \log_{10} H(s)H(-s) \Big|_{s=j\Omega} \quad (II.16)$$

where by definition

$$H(s) = \frac{E(s)}{P(s)} \quad (II.17)$$

H(s) will be called "transducer function". Obviously,

$$\frac{P_{ref}}{P_t} \geq 1 \quad \text{yields} \quad H(j\Omega) \geq 1 \quad (II.18)$$

Alternatively, with equations (II.14) and (II.15), the transducer loss can be written in the form

$$A_t(dB) = 10 \log \left. \frac{F(s)F(-s) + P(s)P(-s)}{P(s)P(-s)} \right|_{s=j\Omega}$$

$$A_t(dB) = 10 \log_{10} (1 + K(s)K(-s)) \Big|_{s=j\Omega} \quad (II.19)$$

where by definition

$$K(s) = \frac{F(s)}{P(s)} \quad (II.20)$$

K(s) will be called "characteristic function".
From equations (II.16) and (II.19) one obtains

$$H(s)H(-s) = 1 + K(s)K(-s) \quad (II.21)$$

or

$$K(s)K(-s) = H(s)H(-s) - 1 \quad (II.22)$$

Any transducer function which satisfies equation (II.18) can always be related to one or several characteristic functions. Obviously, the zeroes of the right side of equation (II.22) form a quadrantal root pattern.

Furthermore, roots on the j-axis must have even multiplicity because:

$$H(j\Omega) * H(-j\Omega) - 1 \geq 0$$

Therefore, it is possible to separate the zeroes into two sets which are the image of each other with respect to the imaginary axis. This separation may be possible in more than one way. There are as many Characteristic Functions as there are ways to select different sets of roots for K(s). The conditions imposed on the Transfer Polynomials in combination with equation (II.18) are also sufficient to assure a realizable input impedance Z.

Proof ([CA-1], pg.427)

F(s)/E(s) is analytic in the entire right half plane including the j-axis as the boundary. Therefore, according to the maximum modulus theorem, its maximum must occur on the boundary. An upper limit for this maximum is obtained from equations (II.14) and (II.18) in the following way:

$$1 = \frac{F(s)F(-s) + P(s)P(-s)}{E(s)E(-s)} \Big|_{s=j\Omega}$$

$$\frac{F(s)F(-s)}{E(s)E(-s)} \Big|_{s=j\Omega} = 1 - \frac{P(s)P(-s)}{E(s)E(-s)} \Big|_{s=j\Omega} = 1 - \frac{1}{H(s)H(-s)} \Big|_{s=j\Omega} \leq 1$$

Consequently,

$$\operatorname{Re}\left(\frac{1 - \dfrac{F(s)}{E(s)}}{1 + \dfrac{F(s)}{E(s)}}\right) \Big|_{s=j\Omega} = \operatorname{Re}\left(\frac{E(s) - F(s)}{E(s) + F(s)}\right) \Big|_{s=j\Omega} = \operatorname{Re}\big(z(s)\big) \Big|_{s=j\Omega}$$

z(s) satisfies therefore all conditions for a positive real function and is realizable.

For later use it is also necessary to express the quantities V_1 and I_1 from Fig. II.1 part B, in terms of H(s) and K(s):

$$V_1 = V_0 \frac{Z}{R+Z} = \frac{V_0}{2} \frac{2Z}{R+Z} = V_{ref} \frac{2z(s)}{1+z(s)}$$

$$I_1 = V_0 \frac{1}{R+Z} = \frac{V_0}{2R} \frac{2R}{R+Z} = I_{ref} \frac{2}{1+z(s)}$$

Substituting the expression of equation (II.10) for the normalized impedance z(s) yields

$$V_1 = V_{ref}\,\frac{E(s)-F(s)}{E(s)} = V_{ref}\,\frac{H(s)-K(s)}{H(s)} \tag{II.23}$$

$$I_1 = I_{ref}\,\frac{E(s)+F(s)}{E(s)} = I_{ref}\,\frac{H(s)+K(s)}{H(s)} \tag{II.24}$$

In these expressions, V_{ref} and I_{ref} are reference quantities according to Fig. II.1, part A.

III. The Power Transfer through the Transmission Network

After having established the quantities V_1 and I_1 across an arbitrary Z, the next logical step is to consider $Z = Z_{in}$ to be the input impedance of a transmission network (see Fig. III.1). In this figure, the generator is represented by V_o and R_1 and delivers the power P_t into the transmission network. In case of lossless transmission networks this power can only be dissipated in the load resistor R_2. Therefore,

$$P_t = P_2\,\frac{|V_2|^2}{R_2}$$

and

$$A_t\ (dB) = 10\ \log_{10}\frac{P_{ref}}{P_t} = 10\ \log_{10}\frac{P_{ref}}{P_2}$$

$$A_t\ (dB) = 10\ \log_{10}\frac{|V_o|^2}{4R_1}\,\frac{R_2}{|V_2|^2} = 10\ \log_{10}\left|\frac{V_0}{2V_2}\right|^2 + 10\ \log_{10}\frac{R_2}{R_1} \tag{III.1}$$

For specified transfer properties of the transmission network in Fig. III.1 the network must have the proper input impedance. According to previous considerations, such an impedance can be designed from suitable selected functions H(s) or K(s). Once this is accomplished there are a number of ways of realizing the resulting impedance. The realization methods by Bott-Duffin and Miata ([VA-1] pgs. 173-182) must be ruled out because the resulting networks contain in general several resistors which dissipate the input power. Suitable is the realization method by Darlington ([DA-1]) which yields the configuration of Fig. III.1. Also suitable is the method by Brune. Both methods will be discussed in detail.

1. <u>The two–port parameters of the transmission Network (Darlington Method)</u>

The object of this subsection is to derive the two–port parameters of the transmission network and to express them in terms of the transfer polynomia1s. The starting point is equation (11.10) for the normalized impedance. It can be written in the form:

$$z_{in}(s) = \frac{E(s) - F(s)}{E(s) + F(s)} = \frac{(E_e - F_e) + (E_o - F_o)}{(E_e + F_e) + (E_o + F_o)} = \frac{m_1 + n_1}{m_2 + n_2} \qquad \text{(III.2)}$$

where the indices "e" and "o" indicate the respective even and odd parts of E(s) and F(s), and m1 and m2 the even parts, n1 and n2 the odd parts of the numerator and denominator, respectively.

For z_{in} in the network of Fig. III.2, in which all impedances are normalized, the following expressions can be established:

$$z_{in}(s) = \frac{z_{11} + z_{11}z_{22} - z_{12}^2}{1 + z_{22}} = \frac{z_{11} + \dfrac{z_{11}}{y_{22}}}{1 + z_{22}} = z_{11} * \frac{1 + \dfrac{1}{y_{22}}}{1 + z_{22}} \qquad \text{(III.3)}$$

In order to relate equation (III.2) to (III.3) its terms can be rearranged in two ways:

$$z_{in} = \frac{m_1}{n_2} * \frac{1 + \dfrac{n_1}{m_1}}{1 + \dfrac{m_2}{n_2}} \quad \text{or} \quad z_{in} = \frac{n_1}{m_2} * \frac{1 + \dfrac{m_1}{n_1}}{1 + \dfrac{n_2}{m_2}} \qquad \text{(III.4)}$$

By identifying related terms of equation (III.3) and (III.4) one obtains the following mixed set of two-port parameters:

	z11	z22	y22
case A	m1/n2	m2/n2	m1/n1
Case B	n1/m2	n2/m2	n1/m1

According to their structure, z22 and y22 are necessarily realizable reactance functions because the numerators and denominators of these functions are the even and odd parts of a Hurwitz polynomial. The expressions for z11 are composed of the even and odd parts of two different Hurwitz polynomials. However, it can be easily shown that they must also be realizable reactances as long as zin is positive real,([VA–l], pgs. 402–407).

z_{11}	$= \dfrac{E_o - F_o}{E_e + F_e}$	$= \dfrac{E_e - F_e}{E_o + F_o}$
z_{22}	$= \dfrac{E_o + F_o}{E_e + F_e}$	$= \dfrac{E_e + F_e}{E_o + F_o}$
y_{22}	$= \dfrac{E_o - F_o}{E_e - F_e}$	$= \dfrac{E_e - F_e}{E_o - F_o}$
$\lvert z \rvert$	$= z_{11}z_{22} - z^2_{12} = \dfrac{E_e - F_e}{E_e + F_e}$	$= \dfrac{E_o - F_o}{E_o + F_o}$
y_{11}	$= \dfrac{z_{22}}{\lvert z \rvert} = \dfrac{E_o + F_o}{E_e - F_e}$	$= \dfrac{E_e + F_e}{E_o - F_o}$
z_{12}	$= \dfrac{\sqrt{(E_o-F_o)(E_o+F_o)-(E_e-F_e)(E_e+F_e)}}{E_e + F_e}$	$= \dfrac{\sqrt{(E_e-F_e)(E_e+F_e)-(E_o-F_o)(E_o+F_o)}}{E_o + F_o}$
	the numerator $\sqrt{-[E(s)E(-s) - F(s)F(-s)]}$ $= \sqrt{-\ [\pm P^2(s)]}$ is a perfect square if <u>P(s) is an odd function</u> Thus:	$\sqrt{E(s)E(-s) - F(s)F(-s)}$ $= \sqrt{+\ [\pm P^2(s)]}$ is a perfect square if <u>P(s) is an even function</u> Thus:
z_{12}	$= \dfrac{P(s)}{E_e + F_e}$	$= \dfrac{P(s)}{E_o + F_o}$
$-y_{12}$	$= \dfrac{P(s)}{E_e - F_e}$	$= \dfrac{P(s)}{E_o - F_o}$
a_{11}	$= \dfrac{z_{11}}{z_{12}} = \dfrac{E_o - F_o}{P(s)}$	$= \dfrac{E_e - F_e}{P(s)}$
a_{12}	$= \dfrac{\lvert z \rvert}{z_{12}} = \dfrac{E_e - F_e}{P(s)}$	$= \dfrac{E_o - F_o}{P(s)}$
a_{21}	$= \dfrac{1}{z_{12}} = \dfrac{E_e + F_e}{P(s)}$	$= \dfrac{E_o + F_o}{P(s)}$
a_{22}	$= \dfrac{z_{22}}{z_{12}} = \dfrac{E_o + F_o}{P(s)}$	$= \dfrac{E_e + F_e}{P(s)}$

Figure III. 3 Table for the Two-Port Parameters for Equally Terminated Transmission Networks.

The next question which must be answered is: are these mixed sets of two– port parameters compatible? This question can be answered by finding the expression for the missing parameter z12. One of the relations between two–port parameters, namely:

$$\det(z) = z_{11}z_{22} - z_{12}^2 = \frac{z_{11}}{y_{22}} \qquad \text{(III.5)}$$

Yields for case A

$$\frac{m_1}{n_1} * \frac{m_2}{n_2} - z_{12}^2 = \frac{n_1}{n_2}$$

Thus $z_{12} = \frac{1}{n_2}\sqrt{m_1 m_2 - n_1 n_2}$ (III.6)

In the radicand of equation (III.6) one may substitute the corresponding terms in equation (III.2) and obtain

$$\sqrt{m_1 m_2 - n_1 n_2} = \sqrt{(E_e - F_e)(E_e + F_e) - (E_o - F_o)(E_o + F_o)} = \sqrt{E(s)E(-s) - F(s)F(-s)}$$

And in combination with equation (II.13)

$$\sqrt{m_1 m_2 - n_1 n_2} = \sqrt{P(s)P(-s)} \qquad \text{(III.7)}$$

In order to arrive at a rational function for z_{12} ,P(s)P(–s) must be a perfect square which is only possible if P(s) is even. In a completely analogous manner the case B leads to the postulate that P(s) must be an odd function. The z–parameters in both cases may be used to find the expressions for other sets of two–port parameters. Some of these are compiled in the table of Fig. III.3. According to the derivations, these parameters are related to equally terminated transmission networks as shown in Fig. III.4.This fact must be emphasized.

2. Numerical examples

A rather simple first example may now serve as an introduction to the numerical procedure. The resulting transmission network will be used to demonstrate some important design concepts.

(a) Specifications

Design a transmission network for a 600 ohm spectrum generator to pass its fundamental frequency f_1 = 12 kHz and to suppress the second harmonic f_2 = 24 kHz.

(b) Normalization

Arbitrarily, let $f_{ref} = f_1$ = 12 kHz and $R_{ref} = R_{gen}$ = 600 ohms. The

normalized frequencies are then

$\Omega_1 = 1 \rightarrow s_1 = j$ for passing of f_1 and

$\Omega_2 = 2 \rightarrow s_2 = 2j$ for suppressing of f_2.

(c) Characteristic function K(s)

According to equation (II.19), the transmission loss into and through the network is

$$A_t \text{ (dB)} = 10 \log_{10} [1 + K(s)K(-s)]_{s=j\Omega}$$

By setting $$K(s) = C \frac{(s^2 + \Omega_1^2)}{(s^2 + \Omega_2^2)} = C \frac{s^2 + 1}{s^2 + 4},$$

the transmission loss will be 0 at $\Omega = 1$ (i.e. at 12 kHz, eventually), and ∞ at $\Omega = 2$ (i.e. at 24 kHz, eventually). In this example the constant C is completely arbitrary. It may be calculated such that at any specified frequency f_o (except f_1 and f_2, of course) the transducer loss assumes a specified value $A_o = 1.0$ dB at $\Omega = 0$. Then

$$C = \frac{s^2 + 4}{s^2 + 1} \sqrt{10^{0.1A_o} - 1} \,\Bigg|_{s=0} = 2.03$$

and $$K(s) = 2.03\left(\frac{s^2 + 1}{s^2 + 4}\right)$$ from which one may identify

$$F(s) = (s^2 + 1) \text{ and } P(s) = 2.03^{-1} (s^2 + 4) = 0.4926 (s^2 + 4)$$

To attach the constant C to the polynomial P(s) is strictly a matter of choice.

By means of the established K(s) the transducer loss can be calculated over any part of the frequency range. For $\Omega \rightarrow \infty$ it approaches 7.09 dB (see Fig. III.5).

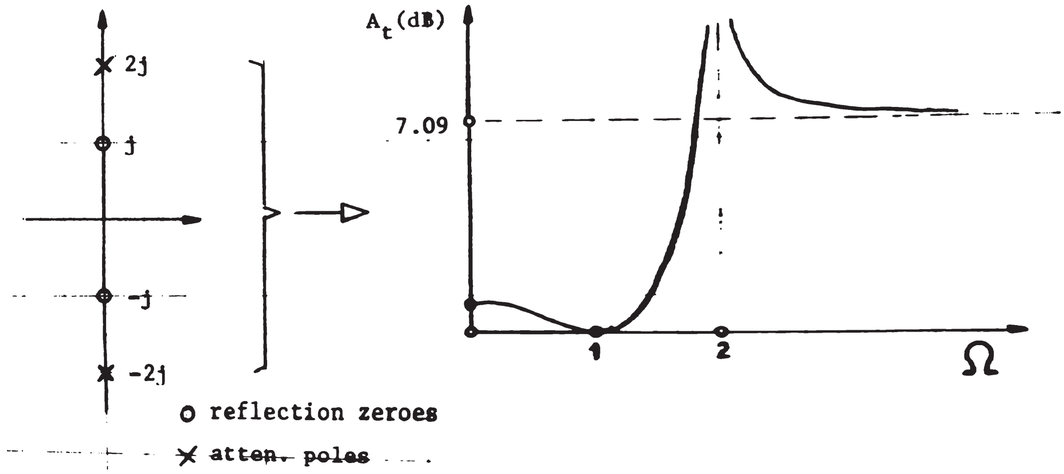


Figure III.5 Pole-zero Pattern for K(s) and related Transducer Loss

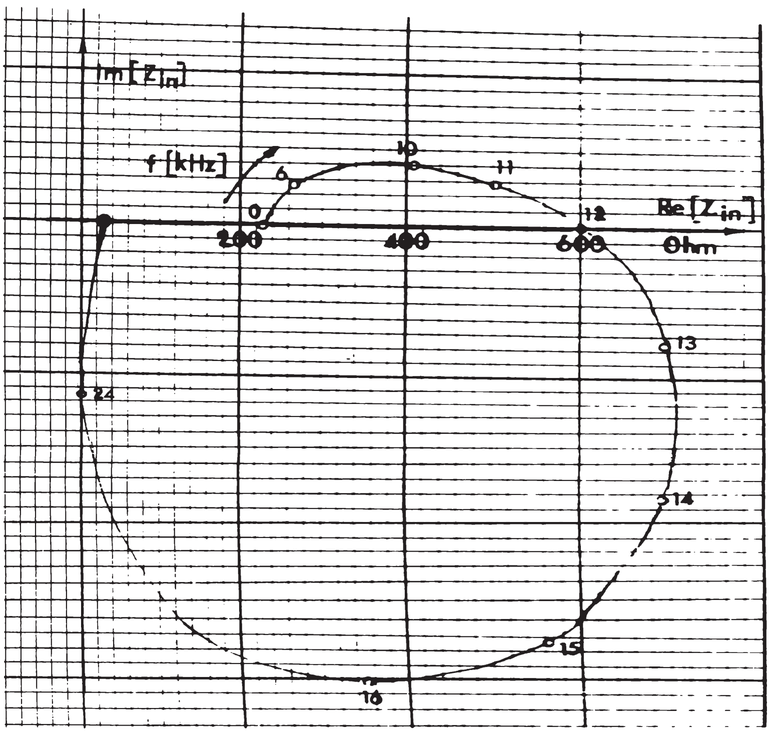


Figure III.6 Locus Curve of $z_{in}(s)$ of the first example

(d) Compatibility equation and H(s)

The next step is to calculate the third transfer polynomial E(s) by means of equation (II.14).

$$E(s)\,E(-s) = (s^2+1)^2 + 0.4926^2(s^2+4)^2$$
$$= 1.2426(s^4 + 3.171\,s^2 + 3.929)$$

In more complicated cases, one must find the roots of the polynomial on the right side and assign those in the left half plane to E(s). However, in this case it is simpler to proceed in the following way

$$E(s)\,E(-s) = \underbrace{\sqrt{1.2426}(s^2+ps+q)}_{E(s)}\;\underbrace{\sqrt{1.2426}(s^2-ps+q)}_{E(-s)}$$

From the product of the two parentheses and by comparison of coefficients one obtains

$$q^2 = 3.929 \rightarrow q = 1.982$$
$$2q - p^2 = 3.171 \rightarrow p = 0.8906$$

Therefore,

$$E(s) = 1.1147(s^2 + 0.8906\,s + 1.982)$$
$$E_e = 1.1147(s^2 + 1.982)\;,\; E_o = 0.9927\,s$$
$$H(s) = \frac{E(s)}{C^{-1}(s^2+4)} = \frac{1.1147(s^2+0.8906s+1.982)}{C^{-1}(s^2+4)}$$
$$H(s) = 2.2628\,\frac{s^2 + 0.8906\,s + 1.982}{(s^2+4)}$$

This such established H(s) is sufficient to realize the transmission network as an active-RC device or as a digital filter.

(e) Realization as an LC network

$E_e = 1.1147\ s^2 + 2.2093$ $\qquad E_o = 0.9927s$

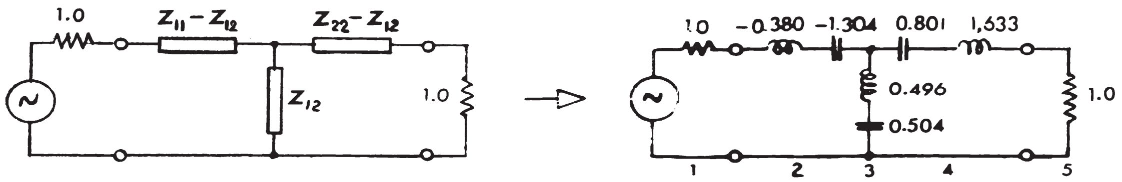
$F_e = s^2 + 1$ $\qquad F_o = 0.0$

With these expressions and the formulas of Fig. III.4, P(s) even, one obtains

$$z_{11} = \frac{E_e - F_e}{E_o} = \frac{0.233s^2+2.455}{2.0153s} \qquad Z_{11}-Z_{12} = -0.380s - \frac{1}{1.304s}$$

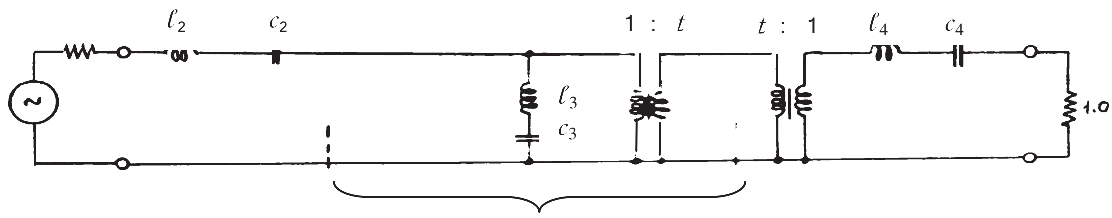
$$z_{12} = \frac{P(s)}{E_o} = \frac{C^{-1}(s^2+4)}{2.0153s} \qquad Z_{11} = 0.496s + \frac{1}{0.504s}$$

$$z_{22} = \frac{E_e + F_e}{E_o} = \frac{4.293s^2 + 6.515}{2.0153s} \qquad z_{22}-z_{12} = 1.633s + \frac{1}{0.801s}$$

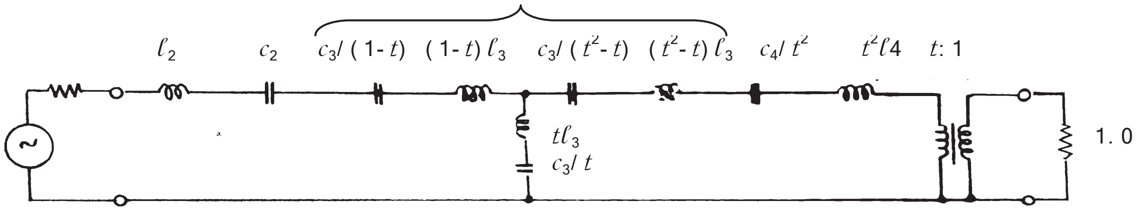


In the right circuit, numbers below indicate the branch numbers

Elimination of negative circuit elements



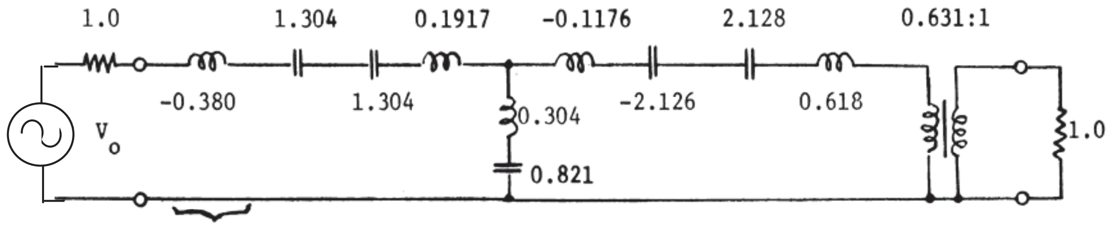
Norton Transformation (See App. A)

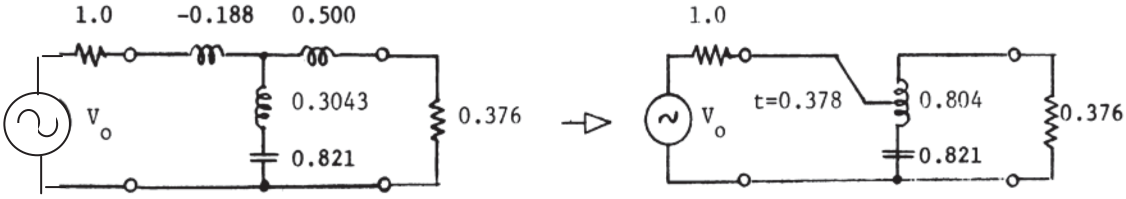


Postulating $C_2 + C_3/(1-t) = 0$ yields $(1-t) = C_3/(-C_2) = 0.504/1.304 = 0.3865$

$$t = 0.6135 \quad , \quad t^2 = 0.3764 \quad , \quad (t^2-t) = -0.2371$$

With these numerical values the circuit becomes





(f) Denormalization

$$f_{ref} = 12 \text{ kHz} \quad , \quad \omega_{ref} = 75.4.10^3 \text{ sec}^{-1} \quad , \quad R_{ref} = 600 \text{ Ohm}$$

$$L_{ref} = \frac{R_{ref}}{\omega_{ref}} = \frac{600}{75.4} \text{ mH} \quad , \quad C_{ref} = \frac{1}{\omega_{ref} R_{ref}} = 22.1 \text{ nF}$$

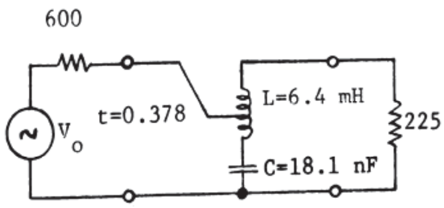
Therefore the actual circuit

$R_{gen} = 600\Omega$

$L = 0.804 \; L_{ref} = 6.4$ mH

$C = 0.821 \; C_{ref} = 18.1$ nF

$R_2 = 0.376 \; R_{ref} = 225 \; \Omega$



(g) Discussion of the results

This rather simple example shows some features which are directly related to the particular pole/zero pattern of the characteristic function. These features are significant even for more complicated circuits. Significant for the above circuit is the appearance of negative elements and the necessity for an ideal transformer if equal terminations are postulated.

In the original circuit, the negative capacitor C_2 is eliminated by a Norton transformation with t O.635. As a result both capacitors in the series branches disappeared. This is not coincidental. Any remaining capacitor in the series branches would cause an attenuation pole at s=0 contrary to the structure of K(s) and H(s). Another significant feature of the circuit is that the triplet of inductors, either before or after the transformation, satisfies the condition for perfect coupling, namely

$$\ell_a\ell_b + \ell_a\ell_c + \ell_c\ell_b = 0 \tag{III.8}$$

$$\ell_a = -0.188 \quad \ell_b = 0.3043 \quad \ell_c = 0.50$$

For s→•, this triplet of inductors approaches the performance of an ideal transformer. If equation (III.8) were not satisfied it would approach an ideal transformer plus some leakage inductance in one or both series branches and, therefore, cause an attenuation pole at •. This would be also contrary to the structure of K(s) and H(s).

At s = 0, i.e. for dc , the final circuit represents a through connection between the generator and the load. However, the finite loss of 1.0 dB at s = O requires either the insertion of an ideal transformer or R_{gen} • R_{load}. If the reflection zero were placed at the origin rather than 12kHz, i.e. if the numerator of K(s) contained the factor s^2 rather than $(s^2 + 1)$, one can predict $R_{gen} = R_{load}$, without the need for an ideal transformer. On the other hand, from equations (II.5, II.7 and II.10) in combination with A = 1.0 dB, postulated, one can predict that the normalized termination must either be 0.376 or its reciprocal. In this rather simple example, it is also possible to predict the tap ratio t= 0.378 of the inductance L in the final circuit. For , the capacitor becomes a short circuit and the transmission network acts as an ideal transformer.

$\lim z_{in} = 0.376\ t^2 = 0.054$

$s \to \infty$

Again, this value can be predicted from $A_t(\infty)$ = 7.09 dB.

The same transmission network can also be obtained by the realization procedure according to Brune ([VA-1], pp 164-172).

According to equation (II.10)

$$z_{in}(s) = \frac{E(s)+F(s)}{E(s)-F(s)} = \frac{0.1147s^2 + 0.9927s + 1.2093}{2.1147s^2 + 0.9927s + 3.2093}$$

By simple calculations,

$$z_{in}(0) = 0.376, \quad z_{in}(j) = \frac{1.0946+0.9927j}{1.0946+0.9927j} = 1 \quad \text{(refl.zero)}, \quad z_{in}(\infty) = 0.054$$

Furthermore, at the attenuation pole

$$z_{in}(2j) = -\frac{0.7505 + 1.985j}{5.2495 + 1.985j} = j\,\frac{0.7505}{1.985}\,\frac{(1+2.645j)}{(1+2.644j)} = 0.378j$$

pure imaginary. Therefore, the resulting conventional Brune section will contain no series resistor. It will be shown later that this is true in general as long as $P(s)$ is either even or odd. The locus curve of $z_{in}(s)$ in the s-plane is shown in Fig. III.6.

(h) Exercises

For the following Characteristic Functions of 2nd degree

$$K(s) = C\frac{s^2}{s^2+1} \quad , \quad K(s) = C\frac{s^2+1}{s^2} \quad , \quad K(s) = Cs^2 \quad , \quad K(s) = C\frac{s^2+1}{s^2-1}$$

(1) determine the po1e—zero pattern in the s-plane

(2) select a suitable constant and calculate the transducer loss at some significant frequencies.

(3) carry out the rea1ization.

For the novice in the synthesis field, it is a valuable and convincing experience not only to carry out once the calculations by hand but also to construct and test the network.

(i) Practical considerations

The resulting circuit from the first example is a so—called canonical circuit because the transmission network contains the least number of elements (counting a tapped coil as one element). In spite of this economy in elements, designers quite often avoid sections of this type. The major reason is that the actual performance deteriorates rather rapidly as the leakage inductance of the tapped coil increases ([VA—1], pp. 130—182). However, for modern components (like pot cores), and tap ratios in the range O.6...1.0, the leakage is very small and these sections are quite practical. A final remark: by a proper choice of A(0), one can enforce a specified termination. For instance, to enforce R2 = 150 Ohms

$$\rho(0) = \frac{600-150}{600+150} = \frac{450}{750} = 0.6$$

$$A(0) = 10\log_{10}\frac{1}{1+\left|\rho(0)\right|^2} = 10\log_{10}1.56 = 1.931dB.$$

p(0) = 600 + = 0.6

A(O) = 10 1og1. 2 = 1og10 1.56 = 1.931 dB.

This attenuation rather than 1.0 dB as above, will result in the desired termination of 150 Ohms.

IV. Voltage and Current Transfer through the Network

As demonstrated in the previous numerical example, the parameters of the z- or y- matrix are the starting point for the realization of the transmission network. Various realization methods will be discussed in greater detail in Section VII. At this point, the main objective is to establish the transfer properties of the transmission network, i.e. the relationships between the input and output voltages and currents. To this end, the chain matrix is the most convenient tool. Also, from the structure of this matrix the various network types are most easily identified.

1. The transfer function between source and load voltages

The two sets of parameters of the A-matrix for case A and B can easily be combined into only one set by making use of the following identities:

	P(s)= even	P(s)=odd
$H_e =$	$E_e/P(s)$	$E_o/P(s)$
$H_o =$	$E_o/P(s)$	$E_e/P(s)$
$K_e =$	$F_e/P(s)$	$F_o/P(s)$
$K_o =$	$F_o/P(s)$	$F_e/P(s)$

(IV. 1)

Therefore for both cases,

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix} = \begin{pmatrix} (H_e-K_e) & (H_o-K_o) \\ (H_o+K_e) & (H_e+K_e) \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix} \qquad \text{(IV.2)}$$

or in the inverse form

$$\begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = \begin{pmatrix} a_{22} & a_{12} \\ a_{21} & a_{11} \end{pmatrix} \begin{pmatrix} V_1 \\ -I_1 \end{pmatrix} = \begin{pmatrix} (H_e+K_e) & (H_o-K_o) \\ (H_o+K_o) & (H_e-K_e) \end{pmatrix} \begin{pmatrix} V_1 \\ -I_1 \end{pmatrix} \qquad \text{(IV.3)}$$

These matrix equations relate the input and output quantities of the normalized circuit in Fig. III.4. For the de-normalized circuit, the quantities a12 and a21 must be multiplied by R_{ref} and $_{Rref-1}$, respectively. Considering also that V2=-RI2, the de-normalized equations (IV.2) become:

$$V_1 = (H_e - K_e)V_2 - (H_o - K_o)RI_2 = [(H_e + H_o) - (K_e + K_o)]V_2 = [H(s) - K(s)]V_2$$

$$I_1 = (H_o + K_o)V_2 - (H_e + K_e)RI_2 = -[(H_e + H_o) + (K_e + K_o)]I_2 = [H(s) + K(s)](-I_2)$$

Therefore in Fig. III.4, the transducer functions of the voltages and currents at the terminals, $H_v(s)$ and $H_i(s)$ respectively, are :

$$H_v(s) = \frac{V_1}{V_2} = H(s) - K(s) \qquad \text{(IV.4)}$$

$$H_i(s) = \frac{I_1}{-I_2} = H(s) + K(s) \qquad \text{(IV.5)}$$

Combining these with equations (II.23) and (II.24) relates V_2 and I_2 to V_{ref} and I_{ref} of Fig. II.1.

$$H(s) = \frac{V_{ref}}{V_2} = \frac{1/2V_0}{V_2} = \frac{I_{ref}}{-I_2} \qquad \text{(IV.6)}$$

The power loss A_2(dB) through the network is obviously

$$A_2(dB) = 10\log\frac{P_{ref}}{P_2} = 10\log\left|\frac{V_{ref}}{V_2}\right|^2 = 10\log\left|H(s) * H(-s)\right|_{s=j\Omega} \qquad \text{(IV.7)}$$

Which is necessarily equal to At(dB) as defined previously. In the chain matrix of reactive networks, the terms of the main diagonal are always even, the other terms odd functions of s. Furthermore from eq. (IV.2),

$$H(s) = \frac{1}{2}\left[(a_{22} + a_{21}) + (a_{12} + a_{11})\right] \qquad \text{(IV.8)}$$

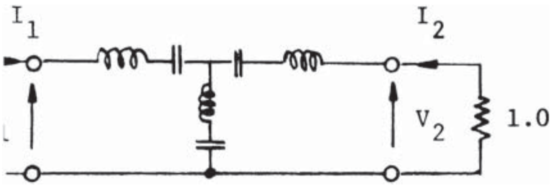
$$K(s) = \frac{1}{2}\left[(a_{22} + a_{21}) - (a_{12} + a_{11})\right] \qquad \text{(IV.9)}$$

If the network were reversed, the terms a_{11} and a_{22} are interchanged. This of course does not alter eq. (IV.8) and, therefore, the network displays the same transfer properties as in the forward direction.

In the previous example, subsection II.2,

$$K(s) = 2.03\,\frac{s^2+1}{s^2+4}\ ,\ H(s) = 2.268\,\frac{s^2+0.8906s+1.982}{s^2+4}$$

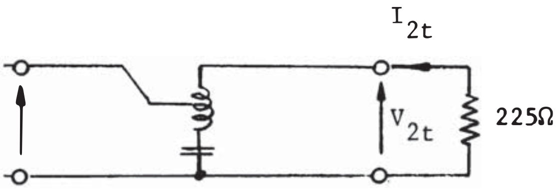
Therefore, according to equations (IV.4) and (IV.5)



$$H_v = \frac{V_1}{V_2} = \frac{0.238s^2+2.019s+2.465}{s^2+4}$$

$$H_i = \frac{I_1}{-I_2} = \frac{4.298s^2+2.019s+6.525}{s^2+4}$$

In the final circuit, after the insertion of the transformer,



$$H_{vt} = \frac{V_1}{V_{2t}} = tH_v = 0.631\ H_v$$

$$H_{it} = \frac{I_1}{-I_{2t}} = \frac{1}{t}\,H_i = 1.585H_i$$

2. **Phase shift**

For $s=j\Omega$, the transducer function expresses both a magnitude and a phase relation between the source and load voltage. Let

$$V_o = |V_o|\ e^{j\phi_0} \quad \text{and} \quad V_2 = |V_2|\ e^{j\phi_2} \tag{IV.10}$$

then
$$H(j\Omega) = \frac{1/2\ V_0}{V_2} = \left|\frac{1/2V_0}{V_2}\right|\ e^{j(\phi_0-\phi_2)} = \left|\frac{1/2\ V_0}{V_2}\right| e^{j\phi} \tag{IV.11}$$

where
$$\phi = (\phi_0-\phi_2) \tag{IV.12}$$

is the phase difference between the source and the load voltage.

$$H(j\Omega) = H_e\ (j\Omega)+H_o(j\Omega) = H_e(j\Omega)\ \left[1 + \frac{H_o(j\Omega)}{H_e(j\Omega)}\right] \tag{IV.13}$$

Therefore, with the identities (IV.1), for

(a) <u>P(s)is even</u>

$$\phi = \arctan\left[\frac{i}{j}\frac{E_o(j\Omega)}{E_e(j\Omega)}\right] + k\pi \qquad \text{(IV.14)}$$

(b) <u>P(s)is odd</u>

$$\phi = \arctan\left[\frac{i}{j}\frac{E_e(j\Omega)}{E_o(j\Omega)}\right] = arc\cot\left[j\frac{E_o(j\Omega)}{E_e(j\Omega)}\right] = \frac{1}{2\pi} + \arctan\left[\frac{i}{j}\frac{E_o(j\Omega)}{E_e(j\Omega)}\right] + k\pi \qquad \text{(IV.15)}$$

Except for the term 1/2π for P(s) odd, the expressions are identical. It is significant for lossless transmission networks that the phase shift ϕ depends only on the Hurwitz polynomial E(s), i.e. only on the location of the natural modes. It can even be expressed directly by the parameters of the factors of E(s). Let

$$E(s) = \Pi_\mu(s + a_\mu)\Pi_\nu(s^2 + p_\nu s + q\nu) \qquad \text{(IV.16)}$$

Then ([CA-1],p. 264),

$$\phi = \frac{1}{2\pi} + \Sigma_\mu \frac{a_\mu}{\Omega} + \Sigma_\nu \frac{p_\nu}{q_\nu - \Omega^2} + k\pi \qquad \text{(IV.17)}$$

This result is easily obtained by an appropriate factorization of H(s) in equation (IV.l1). Fig. IV.1 shows typical phase shift curves for linear and quadratic root factors.

The necessity of a phase jump at $\Omega = \sqrt{q_\nu}$ can easily be understood by considering the previous example. In this circuit, obviously, input and output are in phase at Ω=0 and Ω=∞. If the phase curve were continuous the voltages would be out of phase by 180° at Ω=∞.

3. <u>Group delay</u>

The normalized group delay τ is defined as the differential quotient of ϕ with respect to Ω; thus

$$\tau = \frac{d\phi}{d\Omega} = \frac{d\phi}{ds}\frac{ds}{d\Omega} = \frac{1}{(1 - \frac{E_o^2}{E_e^2})} * \frac{1}{j} * \frac{E_e E_o' - E_o E_e'}{E_e^2} * j \,\Big|_{s=j\Omega}$$

$$\tau = \frac{E_e(j\Omega)E_o'(j\Omega) - E_o(j\Omega)E_e'(j\Omega)}{E_e^2(j\Omega) - E_o^2(j\Omega)} \qquad \text{(IV.18)}$$

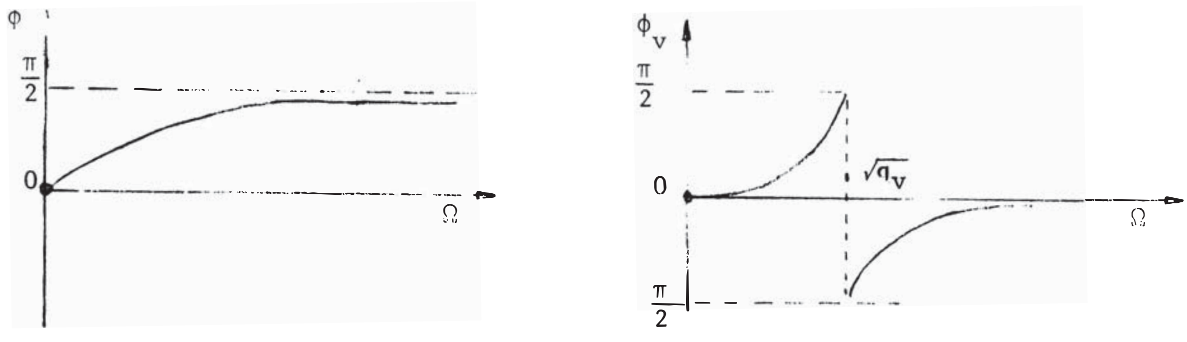


Figure IV. 1 Phase Contributions by Factors of E(s)

Matrix Parameters	K(s) = odd F(s) = odd, P = even	K(s) = odd F(s) = even, P = odd	K(s) = even F(s) = even, P even
A	$\frac{E_e}{P} = H_e$	$\frac{E_o}{P} = H_e$	$\frac{E_e - F_e}{P} = H_e - K_e$
B	$\frac{E_o - F_o}{P} = H_o - K_o$	$\frac{E_e - F_e}{P} = H_o - K_o$	$\frac{E_o}{P} = H_o$
C	$\frac{E_o + F_o}{P} = H_o + K_o$	$\frac{E_e + F_e}{P} = H_o + K_o$	$\frac{E_o}{P} = H_o$
D	$\frac{E_e}{P} = H_e(s)$	$\frac{E_o}{P} = H_e$	$\frac{E_e + F_e}{P} = H_e + K_e$
z_{11}	$\frac{E_e}{E_o + F_o}$	$\frac{E_o}{E_e + F_e}$	$\frac{E_e - F_e}{E_o}$
y_{11}	$\frac{E_e}{E_o - F_o}$	$\frac{E_o}{E_e - F_e}$	$\frac{E_e + F_e}{E_o}$
z_{22}	$\frac{E_e}{E_o + F_o} = z_{11}$	$\frac{E_o}{E_e + F_e} = z_{11}$	$\frac{E_e + F_e}{E_o} = y_{11}$
y_{22}	$\frac{E_e}{E_o - F_o} = y_{11}$	$\frac{E_o}{E_e - F_e} = y_{11}$	$\frac{E_e - F_e}{E_o} = z_{11}$
	Symmetrical networks		Antimetrical networks

Figure IV. 2 Two-Port Parameters of Symmetrical and Antimetrical Networks

The actual group delay is defined as the differential quotient of the phase with respect to $\omega=2\pi f$, thus

$$t = \frac{d\phi}{d(2\pi f)} = \frac{1}{2\pi f_{ref}} * \frac{d\phi}{d\Omega} = T_{ref} * \tau \qquad \text{(IV.19)}$$

Where Tref is the reference group delay defined as

$$T_{ref} = \frac{1}{2\pi f_{ref}} \qquad \text{(IV.20)}$$

4. Numerical example (Subsection III.2)

$E(s) = C(s^2 + 0.8906s - 1.982)$ and P(s) is even

One obtains for the phase shift:

$$\phi = \arctan\left(\frac{1}{j}\cdot\frac{0.8906s}{s^2+1.982}\right)\Bigg|_{s=j} + k\cdot\pi = \arctan\left(\frac{0.8906\Omega}{1.982-\Omega^2}\right) + k\cdot\pi$$

And for the normalized group delay:

$$\tau = \frac{(s^2+1.982)\cdot 0.8906 - 0.8906s\cdot 2s}{\left(s^2+1.982\right)^2 - \left(0.8906s\right)^2}\Bigg|_{s=j} = \frac{0.890\Omega^2 + 1.765}{\Omega^4 - 3.171\Omega^2 + 3.928}$$

To obtain the expression for the actual group delay, one must multiply the normalized delay by T_{ref}.

$$T_{ref} = (2\pi f_{ref})^{-1} = 13.26\ \mu\text{sec.}$$

5. Network types

From equations (IV.2) and (IV.3) one obtains for the normalized driving point impedances

at the input: $z_1 = \dfrac{V_1}{I_1} = \dfrac{(H_e + H_o) - (K_e + K_o)}{(H_e + H_o) + (K_e + K_o)} = \dfrac{H(s) - K(s)}{H(s) + K(s)}$ (IV.21)

at the output: $z_2 = \dfrac{V_2}{-I_2} = \dfrac{(H_e + H_o) + (K_e - K_o)}{(H_e + H_o) - (K_e - K_o)} = \dfrac{H(s) + K(s)}{H(s) - K(s)}$ (IV.22)

In general, these impedances will be related only through a common set of transfer polynomials. However, of practical interest are those cases in which the following relations exist between the terminal impedances:

$$z_1 = z_2 \quad \text{and} \quad z_1 z_2 = 1 \qquad \text{(IV.23)}$$

For these special types, K(s) must either be odd or even.

(a) <u>K(s) = odd function → impedance symmetry</u>

in this case

$$K_e \equiv 0 \ , \ K(-s) = K(s) \qquad \text{(IV.24)}$$

Consequently in the A-matrix, $a_{11} = a_{22}$ and

$$(A) = \begin{pmatrix} H_e & H_o - K_o \\ H_o + K_o & H_e \end{pmatrix} \qquad \text{(IV.25)}$$

Both driving point impedances are

$$z_1 = z_2 = \frac{H(s)-K(s)}{H(s)+K(s)} \quad \frac{E(s)-F(s)}{E(s)+F(s)} \ , \ \rho_1(s) = \rho_2(s) \qquad \text{(IV.26)}$$

K(s) = odd requires either P(s) = odd and F(s) = even or vice versa. Consequently, the compatibility equation (II.14) takes the form

$$E(s)E(-s) = \pm[F(s) + P(s)][F(s)-P(s)] \qquad \text{(IV.27)}$$

The two bracketed expressions on the right side have root patterns with mirror symmetry about the j-axis. Therefore, either root pattern yields the full information regarding the roots of E(s). One merely has to solve, for instance,

$$[F(s) + P(s)] = 0$$

and to transpose its roots in the right half plane to the left.

(b) <u>K(s) = even → antimetrical networks</u>

In this case

$$K_o \equiv 0 \ , \ K(-s) = K(s) \qquad \text{(IV.28)}$$

Consequently in the A-matrix, $a_{12} = a_{21}$, and

$$(A) = \begin{pmatrix} H_e - K_e & H_o \\ H_o & H_e + K_e \end{pmatrix}$$

The driving point impedances are:

$$z_1 = \frac{H(s) - K(s)}{H(s) + K(s)}$$

$z_1 \cdot z_2 = 1$; and de-normalized: $Z_1 \cdot Z_2 = R_{REF}^2$ (IV.30)

$$z_2 = \frac{H(s) + K(s)}{H(s) - K(s)}$$

For the refection coefficient, one obtains:

$\rho_1(s) = \frac{1 - z_1}{1 + z_1} = \frac{K(s)}{H(s)}$ and $\rho_2(s) = \frac{1 - z_2}{1 + z_2} = -\frac{K(-s)}{H(s)}$ (IV.31)

From the special form of the A-matrix, one can easily derive that for z-matrix and the y-matrix:

$$\begin{aligned} \det(z) &= z_{11} \cdot z_{22} - z_{12}^2 = 1 \\ \det(y) &= y_{11} \cdot y_{22} - y_{12}^2 = 1 \end{aligned} \qquad \text{(IV.32)}$$

K(s) being even requires that both F(s) and P(s) are even. Consequently the compatibility equation (II.14) can be written in the form

$E(s) \cdot E(-s) = [F(s) + j(P(s)] \cdot [F(s) - j(P(s)]$ (IV.33)

The two bracketed expressions on the right side have root patterns with symmetry about the origin. Therefore, either root pattern yields again the full information regarding the roots of E(s). One merely has to find those of F(s)+jP(s) and transpose its roots from the right half plane to the left.

(c) Allpass filters

A special type of networks satisfies both the conditions for symmetry and antimetry, i.e. $K_e \equiv 0$ and $K_o \equiv 0$, thus $K(s) \equiv 0$. For these,

$$(A) = \begin{pmatrix} H_e & H_o \\ H_o & H_e \end{pmatrix}, \quad z_1 = z_2 = 1 \tag{IV.34}$$

According to equation (II.21)

$$H(s)\, H(-s)\Big|_{s=j\Omega} = 1 \tag{IV.35}$$

thus

$$H(j\Omega) = e^{j\,\phi\,(\Omega)} \tag{IV.36}$$

These networks will only change the phase but not the magnitude of the signal (allpass filters)

(d) Dual Transmission networks

The dual impedance $Z_1^{(d)}$ of an impedance Z_1 is defined by

$$Z_1^{(d)} = \frac{R_o^2}{Z_1} \tag{IV.37}$$

where R_o is an arbitrary resistor. For $R_o = R_{ref}$, the normalized dual impedance becomes

$$z_1^{(d)} = \frac{1}{z_1} \tag{IV.38}$$

Starting with the dual of z_1 in equation (II.8) yields

$$\rho(s) = \frac{n(s) - d(s)}{n(s) + d(s)} = \frac{-F(s)}{E(s)} \tag{IV.39}$$

All subsequent calculations will be identical except for the reversed sign of F(s) wherever this polynomial appears. For instance, the chain matrix of the dual network is

$$(A^d) = \begin{pmatrix} H_e + K_e & H_o + K_o \\ H_o - K_o & H_e - K_e \end{pmatrix} \tag{IV.40}$$

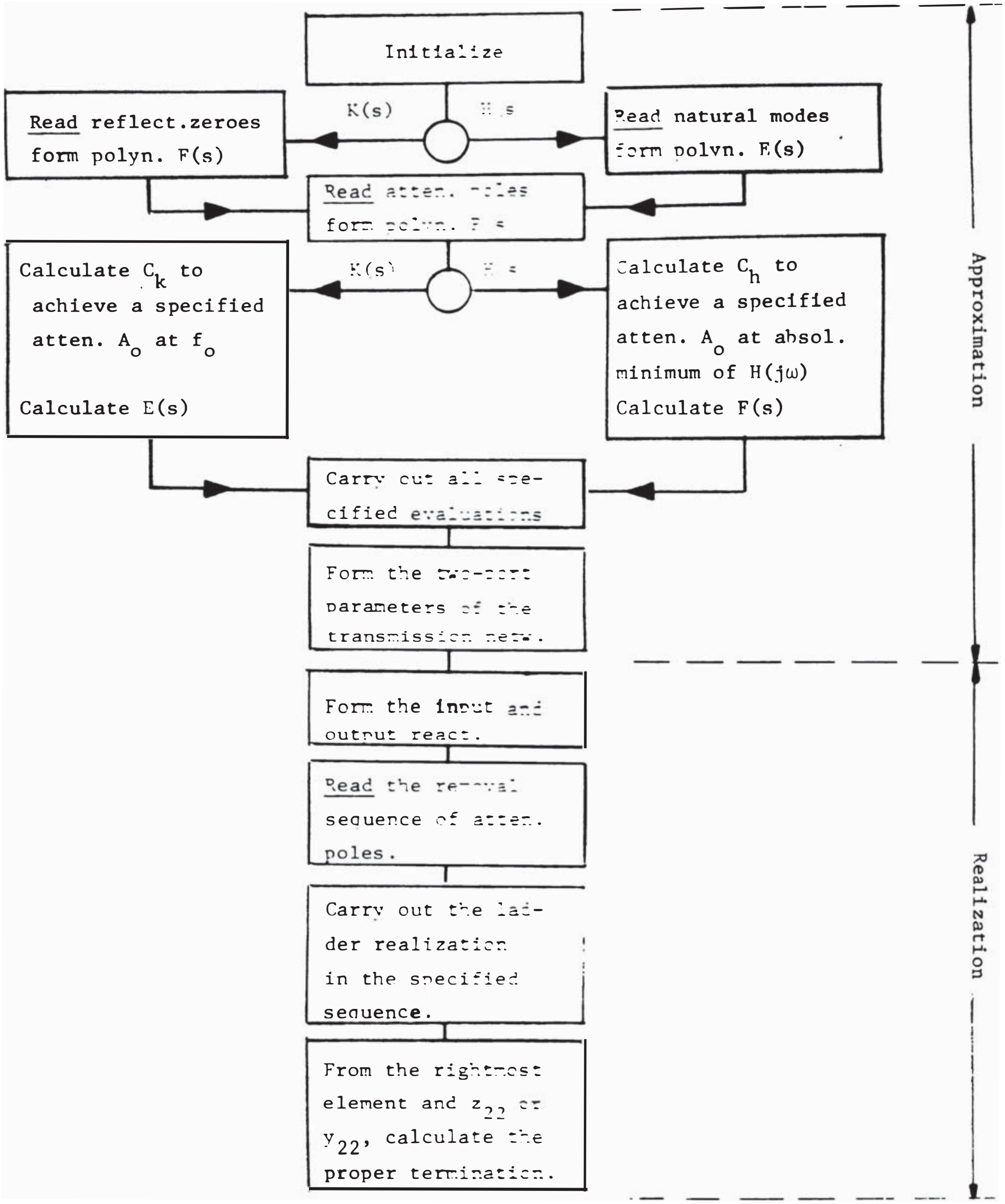


Figure IV. 3 Flowchart of a Synthesis Programm

6. Computer-aided synthesis

In most practical applications, the design of transmission networks follows the same steps which were used in the rather simple example of subsection III.2. However, the calculations become considerably more numerous and their precision more critical as the complexity of the network increases. For these reasons, computers are used extensively in this field.

In the previous example, the specified design parameters were those of the characteristic function. By a suitable selection of its parameters one can tailor the transducer loss to the requirements of a transmission system. Other transfer performances, such as the phase shift, the group delay or any transient response can not be evaluated until the parameters of H(s) have been determined. If the specifications include phase, delay or transient response, it is considerably more practical to use the parameters of H(s) as the starting point of the calculations. However, to find suitable locations for the poles and zeroes of H(s) is considerably more complicated than to find those of K(s).

The procedure by which one finds first a suitable characteristic function K(s) and then a compatible H(s) will be called a "K(s)" design; in the reverse sequence, an "H(s)" design. The consecutive steps for both procedures are shown in the flowchart of Fig. IV.3. A considerable number of steps is identical in both procedures. In a computer program, these steps can be carried out by routines common to both modes. Other calculations which are different for either mode are rather similar in structure. The pertinent routines in the program may be adapted to suit either mode.

A synthesis program of the described type is available from the NC-State University, Department of Electrical Engineering, Raleigh, N.C. it permits the complete design of lossless ladder networks starting either with the parameters of K(s) or H(s). The high degree of freedom for the selection of poles and zeroes makes the program flexible and suitable for unconventional applications. In combination with published parameter tables, it can also be used to solve most conventional applications. Typical examples will be shown below and also in the section for methodical approximation methods.

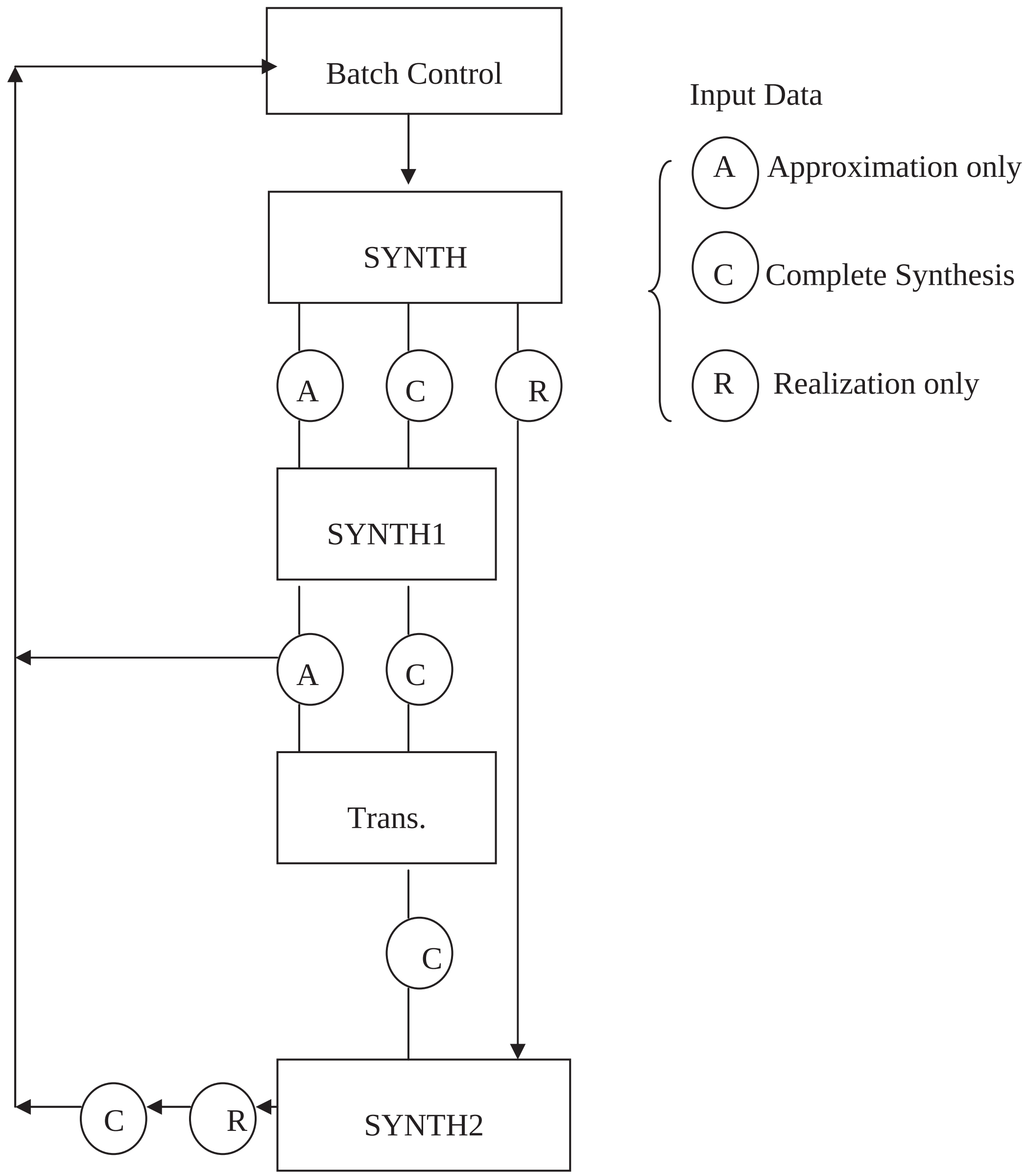


Figure IV.4:Flowchart of N.C.State Synthesis Program

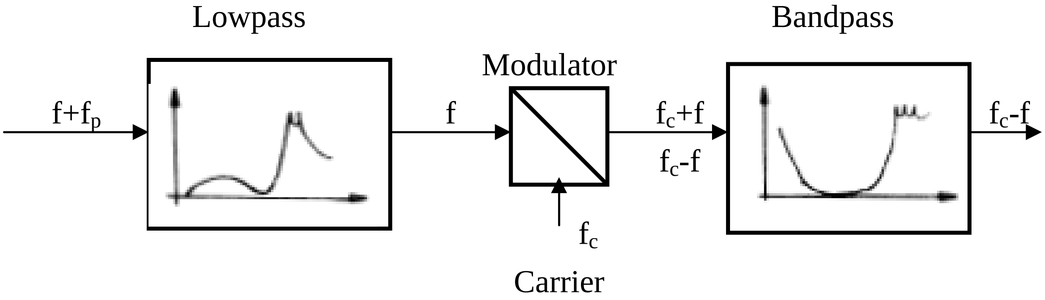
Fig. IV.4 is a flow chart of the major portions of the NC-State synthesis program. Its two essential parts are:

(a) SYNTH1 which contains the calculations of a compatible set of transfer polynomials and the parameters of the A-matrix. It also carries out from one to four evaluations of specified transfer performances. For reference see ([BA-2]).

(b) SYNTH2 which carries out several realizations of ladder circuits for any specified sequence of pole removals. For reference see ([GR-1]). As shown in Fig. IV.4, both parts of the program can be carried out separately and independently, or in sequence. Appendix B describes in detail the data cards for all applications. The program has been written almost entirely in FORTRAN IV for the IBM 360 computer. Only a small portion is in assembler language. At the time this book is written, the program will be available as a service from the CDC Data Center in Stockholm, Sweden.

Example

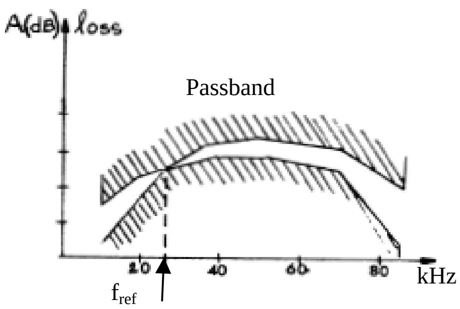
In a single-sideband system, the signal passes first through a lowpass then through a modulator and finally through a bandpass which eliminates the lower sideband. The modulator and the bandpass are fixed

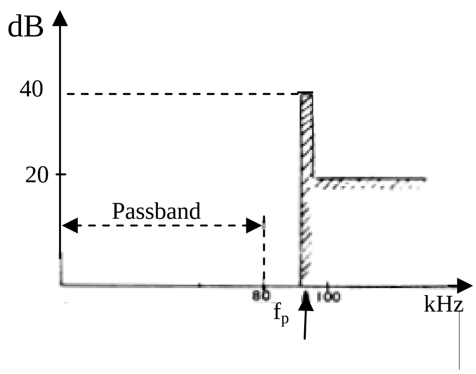


The losses in the bandpass cause an increase in the attenuation at the passband edges: the corners of the passband are rounded rather than sharp. Consequently the low and the high-frequency part of the input signal will be attenuated more. To compensate for these losses, these frequencies are emphasized in the lowpass.

In the stopband, the lowpass must also suppress a single pilot frequency in the range 96-100 kHz by at least 40dB and all frequencies above 100 kHz by at least 20 dB.

The above specifications in combination with tolerances yield the following tolerance plots:





Intuitively, a set of attenuation poles and reflection zeroes was selected for the lowpass. After some improvements by trial and error, the following parameters were chosen:

Reflection zeroes:	-18.75+/- j 80.00 KHz 0.0 +/- j 87.50 KHz	} To emphas. high freq.
	0.0 +/- j 2.50 KHz	To emphas. low freq
Attenuation Poles:	-16.875 KHz	To achieve sharp slope
	1.0 +/- j 87.50 KHz 0.0 +/- j 2.50 KHz	} To achieve 40dB. Atten.

```
TEST FILTER,G.L.BASS NC-STATE UNIV.-EE                                       K(S)-DESIGN

SPECIFICATIONS

DEGREE:  6   REFL.ZEROES       0 AT ORIGIN          3 FINITE PAIRS       0 REAL
             ATTEN.POLES       0 AT ORIGIN          3 FINITE PAIRS       0 QUADR.

REF.FRQ.  25.000 KHZ     A0=    2.500 DB AT F0=    25.000 KHZ            -0.000

EVAL.    LOSS RESPONSE         LOSS RESPONSE        PHASE RESPONSE       STEP RESP.
 FROM        0.000 KHZ          88.000 KHZ            1.000 KHZ            .005 MS
 TO         88.000 KHZ         110.000 KHZ         1000.000 KHZ            .060 MS
 OVER A LINEAR RANGE           LINEAR RANGE         LOGAR.RANGE          LINEAR RANG
 WITH        2.000 KHZ INCR.      .500 FRQ/DEC.      15.000 KHZ INCR.      .001 MS

PLOT MARGINS
 LEFT        0.000 DB            0.000 DB             0.000 DEGR.          -.400 V
 RIGHT      10.000 DB           80.000 DB           750.000 DEGR.          1.600 V
 SUBDIV.     2.500 DB           10.000 DB           180.000 DEGR.           .500 V

REFL.ZER. RE(KHZ)              IM(KHZ)
  X1=     -18.75000000 Y1=     80.00000000
  X2=       0.00000000 Y2=      2.50000000
  X3=       0.00000000 Y3=     87.50000000

ATTEN.PLS RE(KHZ)              IM(KHZ)
  A1=      16.87500000 B1=      0.00000000
  A2=       0.00000000 B2=     96.25000000
  A3=       0.00000000 B3=     99.75000000
```

Part A

```
715  U( 1) = .15515169570932130726503468762D+00     V( 1) = .22103292164540642548056692465D+00     .13363824E-50
600  U( 2) = .15515169570932130726503468763D+00     V( 2) =-.22103292164540642548056692465D+00     .2138211BE-49
516  U( 3) =-.12529826014668108853097658422D+02     V( 3) = .56462779526143769101048420162D+00     .17737552E-47
400  U( 4) =-.12529826014668108853097658437D+02     V( 4) =-.56462779526143769101048422433D+00     .12795807E-47
308  U( 5) =-.10799828312938874480947161167D+02     V( 5) = .57247267194231687183914307393D+01     .12677106E-35

THE FOLLOWING ARE THE NORMALIZED ROOTS OF E(S)E(-S) IN THE Z-PLANE,
     WHERE Z=S**2.

U( 1)= -.107998283129389D+02  V( 1)= -.572472671942317D+01
U( 2)= -.107998283129389D+02  V( 2)=  .572472671942317D+01
U( 3)= -.125298260146681D+02  V( 3)= -.564627795261438D+00
U( 4)= -.125298260146681D+02  V( 4)=  .564627795261438D+00
U( 5)=  .155151695709321D+00  V( 5)= -.221032921645406D+00
U( 6)=  .155151695709321D+00  V( 6)=  .221032921645406D+00

THE FOLLOWING ARE THE NORMALIZED ROOTS OF E(S).

U( 1)= -.843641045180825D+00  V( 1)=  .339286875247970D+01
U( 2)= -.797351075148123D-01  V( 2)=  .354064735635145D+01
U( 3)= -.461087195701989D+00  V( 3)=  .239686683674757D+00
```

Part B

```
THE FOLLOWING ARE THE COEFFICIENTS OF F(S), P(S) AND E(S) LISTED IN ASCENDING ORDER.

F( 0)=   .132330624999997D+01  P( 0)=  -.107516420225154D+03  E( 0)=   .448578925172259D+02
F( 1)=   .183749999999998D+00  P( 1)=  0.                      E( 1)=   .159943819978782D+03
F( 2)=   .132561149999999D+03  P( 2)=   .221968585124998D+03  E( 2)=   .196526457627797D+03
F( 3)=   .183900000000000D+02  P( 3)=  0.                      E( 3)=   .505954228530194D+02
F( 4)=   .230624999999999D+02  P( 4)=   .302869749999999D+02  E( 4)=   .292624978055667D+02
F( 5)=   .150000000000000D+01  P( 5)=  0.                      E( 5)=   .300006557005393D+01
F( 6)=   .100000000000000D+01  P( 6)=   .100000000000000D+01  E( 6)=   .108347598133464D+01

THE CONSTANT ASSOCIATED WITH F(S) AND E(S) IS CONST=  .239786641138525D+01

THE FOLLOWING ARE THE RESIDUES OF 1/H(S) ASSOCIATED WITH A STEP RESPONSE.

RESRE( 0)=  -.999565E+00
RESRE( 1)=  -.302026E+00  RESIM( 1)=   .176243E+00
RESRE( 2)=   .117525E-01  RESIM( 2)=   .457662E-01
RESRE( 3)=   .167475E+01  RESIM( 3)=  -.886873E+00

THE FOLLOWING ARE THE COEFFICIENTS OF THE NUMERATOR OF THE ABCD PARAMETERS.  THE DENOMINATOR
OF EACH OF THESE PARAMETERS IS P(S).

A( 0)=  .435345862672226D+02  B( 0)= 0.                      C( 0)= 0.                      D( 0)=  .461811987672260D+02
A( 1)= 0.                     B( 1)=  .159760069978780D+03  C( 1)=  .160127569978780D+03  D( 1)= 0.
A( 2)=  .639653076277990D+02  B( 2)= 0.                      C( 2)= 0.                      D( 2)=  .329087607627800D+03
A( 3)= 0.                     B( 3)=  .322054228530190D+02  C( 3)=  .689854228530190D+02  D( 3)= 0.
A( 4)=  .619999780556680D+01  B( 4)= 0.                      C( 4)= 0.                      D( 4)=  .523249978055670D+02
A( 5)= 0.                     B( 5)=  .150006557005390D+01  C( 5)=  .450006557005390D+01  D( 5)= 0.
A( 6)=  .834759813346440D-01  B( 6)= 0.                      C( 6)= 0.                      D( 6)=  .208347598133460D+01
```

Part C

Figure IV.5 The Computer Output of SYNTH 1

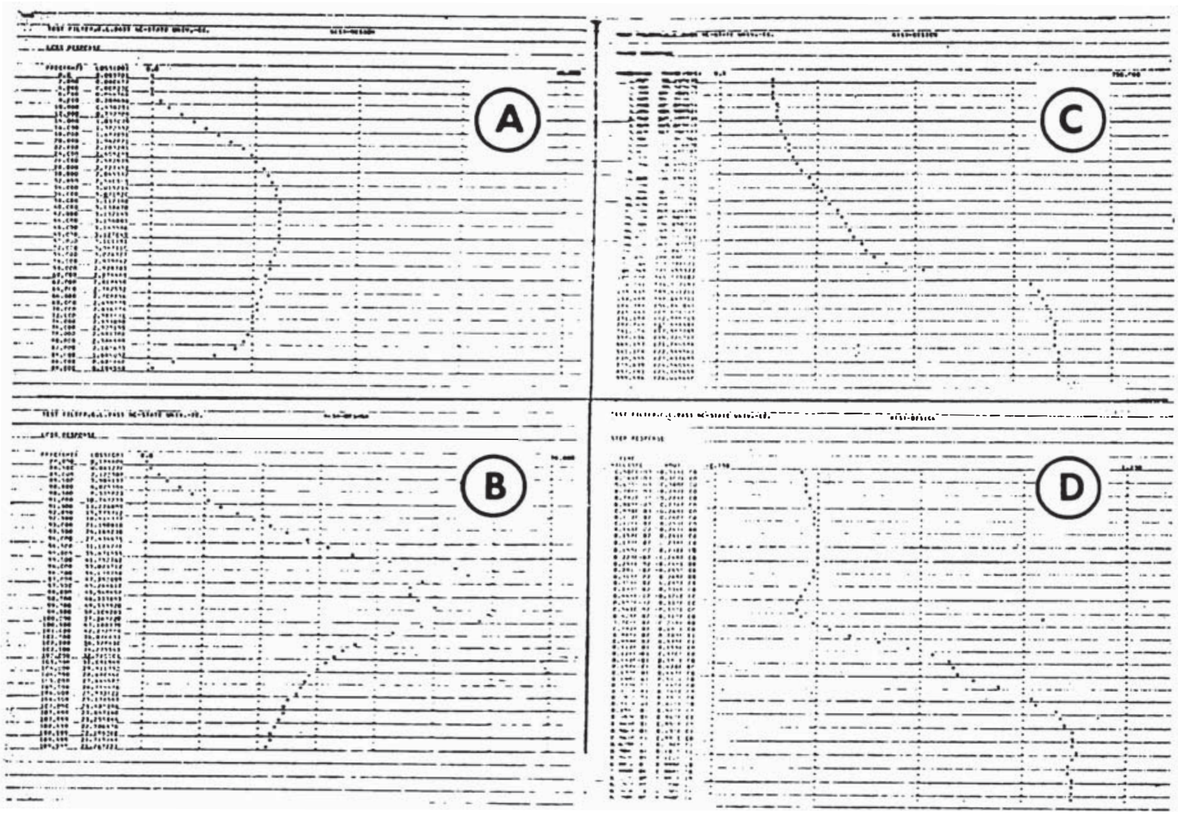


Figure IV.6 The Results of the Specified Loss, Phase and Step Response

(a) Passband Response (c) Phase Response
(b) Stopband Response (d) Step Response

Comments

The reflection zero at 2.5 kHz causes a low loss at the lowest part of the passband. Due to the presence of an attenuation pole not too distant away on the real axis, the attenuation rises sharp as the frequency increases. The loss decreases again as the frequency approaches the vicinity of the two pairs of reflection zeroes at the upper passband edge. For this particular example, the computer output of SYNTH1 is shown in Fig. IV.5 and 6.

In analogy to the previous numerical example in subsection III.2 one may expect

(a) unequal terminations because of the finite loss at $f = 0$

(b) coils with mutual coupling because of the lack of an attenuation pole at infinity,

if the circuit were realized as an LC filter.

This rather unconventional example was selected to demonstrate the heuristic placement of poles and zeroes. Although one would normally use an optimization procedure to achieve a close match it is in most cases necessary or desirable to make reasonable first choices by intuition. One of these optimization methods and also the conventional methods for methodical approximations are subjects of the next section. However, if he so desires the reader may skip directly to the realization procedures.

V. Methodical Approximation Methods

An important part of the design of transmission networks is the methodical selection of the poles and zeroes of K(s) and H(s). The pertinent methods are the subject of that part of synthesis known as "approximation theory". The name "approximation" implies that a specified performance shall be obtained within reasonable and practical limits. The desired performance and the permitted tolerances are most conveniently displayed in a tolerance plot, similar to the one shown in Fig. V.1. Part A of this figure is a typical plot for the transducer loss of bandpass filters. It postulates a constant upper boundary A_{max} for the passband range $f_{-p} \leq f \leq f_p$ and non-constant boundaries $A_{min}(f)$ for the stopband ranges $0 \leq f \leq f_s$ and $f_s \leq f < \infty$. The actual performance of the network (including all deviations due to element inaccuracies and environmental changes) must not cut through the shaded area of the tolerance plot. For this and similar tolerance plots regarding the transducer loss, the characteristic function K(s) is the most convenient starting point. Similar plots may also be prepared for phase and delay and also for various transient responses. Then however, the starting point of the approximation should be the transfer function H(s).

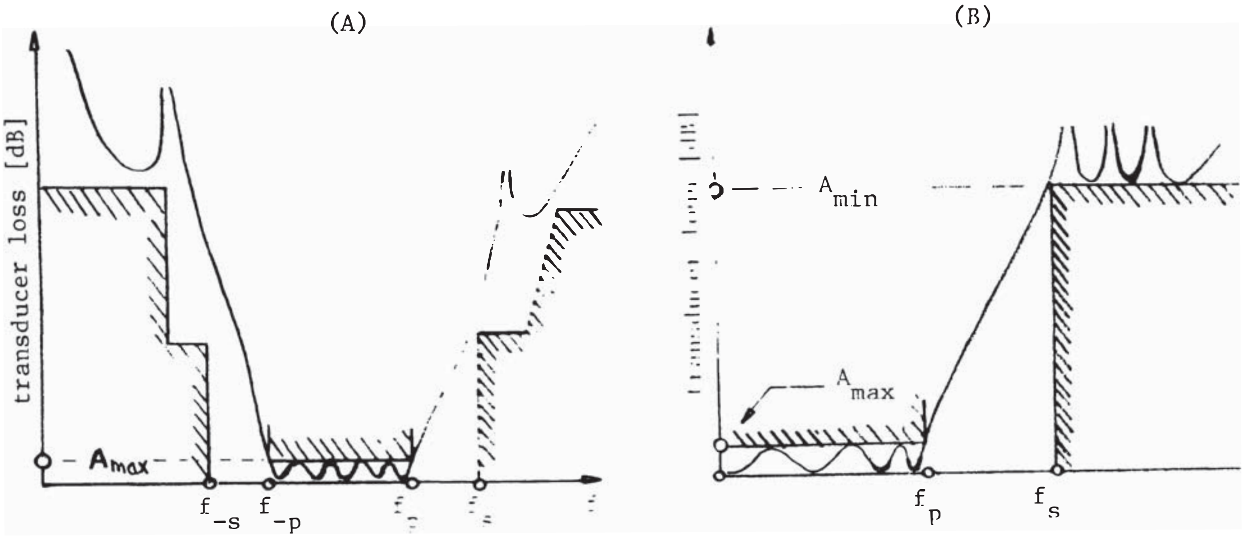


Figure V. 1 Typical Tolerance Plots

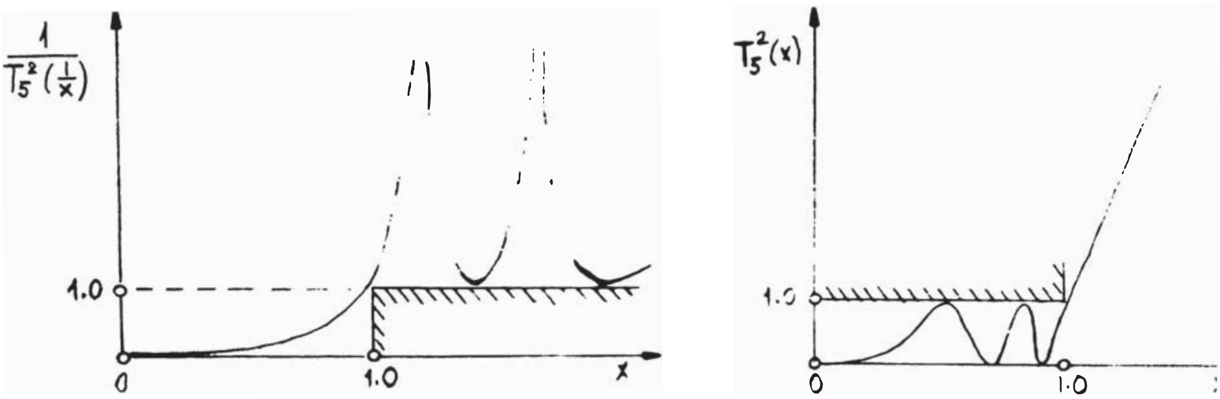


Figure V. 3 Chebyshev Polynomials and their Double Inversion in Squared Form

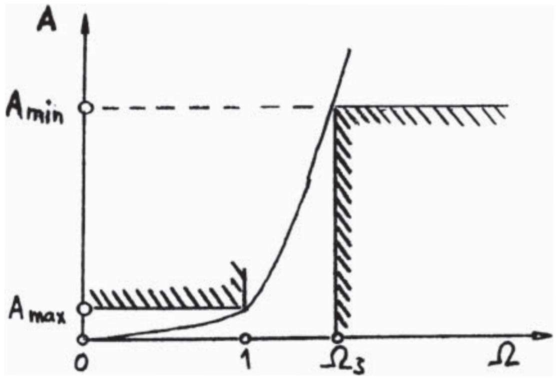
In order to simplify the approximation task, numerous numerical tables have been prepared by many authors, especially during the past decade. A rather comprehensive list of these, compiled by Orchard and Temes, may be found in the IEEE PGCT, Dec. 1968. In combination with the computer program SYNTH, these tables are a considerable design aid. Examples to this end and some of the basic approximation methods are the topics of the following subsections.

1. Classical lowpass approximations

In numerous practical applications, the specifications for the transducer loss postulate a constant upper boundary A_{max} for the passband range and constant lower boundary A_{min} for the stopband or bands. Such simpler tolerance plots may be related to the tolerance plot of a reference lowpass shown in part B, Fig. V.1. The classical solutions for this case are known as lowpass approximations of the Butterworth, Chebyshev, inverted Chebyshev and Cauer-parameter type. Numerous tables exist to aid the deisgn of these transmission networks. Because of the compatibility of the notation with the notation in this chapter, the most suitable references are [CE-1], [SA-1] and [SA-2]. Of these, the parameters tabulated in [CE-1] will be used in the numerical examples.

(a) Butterworth Response

Significant for this type:
all reflection zeroes at the origin,
all attenuation poles at infinity.
Consequently, the loss rises monotonically with rising frequency.



Characteristic function.

$$K(s) = Cs^N \tag{V.1}$$

Transducer Loss

$$A[dB] = 10 \log_{10} [1 + C^2|s|^{2N}]_{s=j\Omega} \tag{V.2}$$

Calculation of C.

$$A[dB] = A_{max} \text{ at } s = j \text{ yields } C = \sqrt{10^{0.1 A_{max}} - 1} \tag{V.3}$$

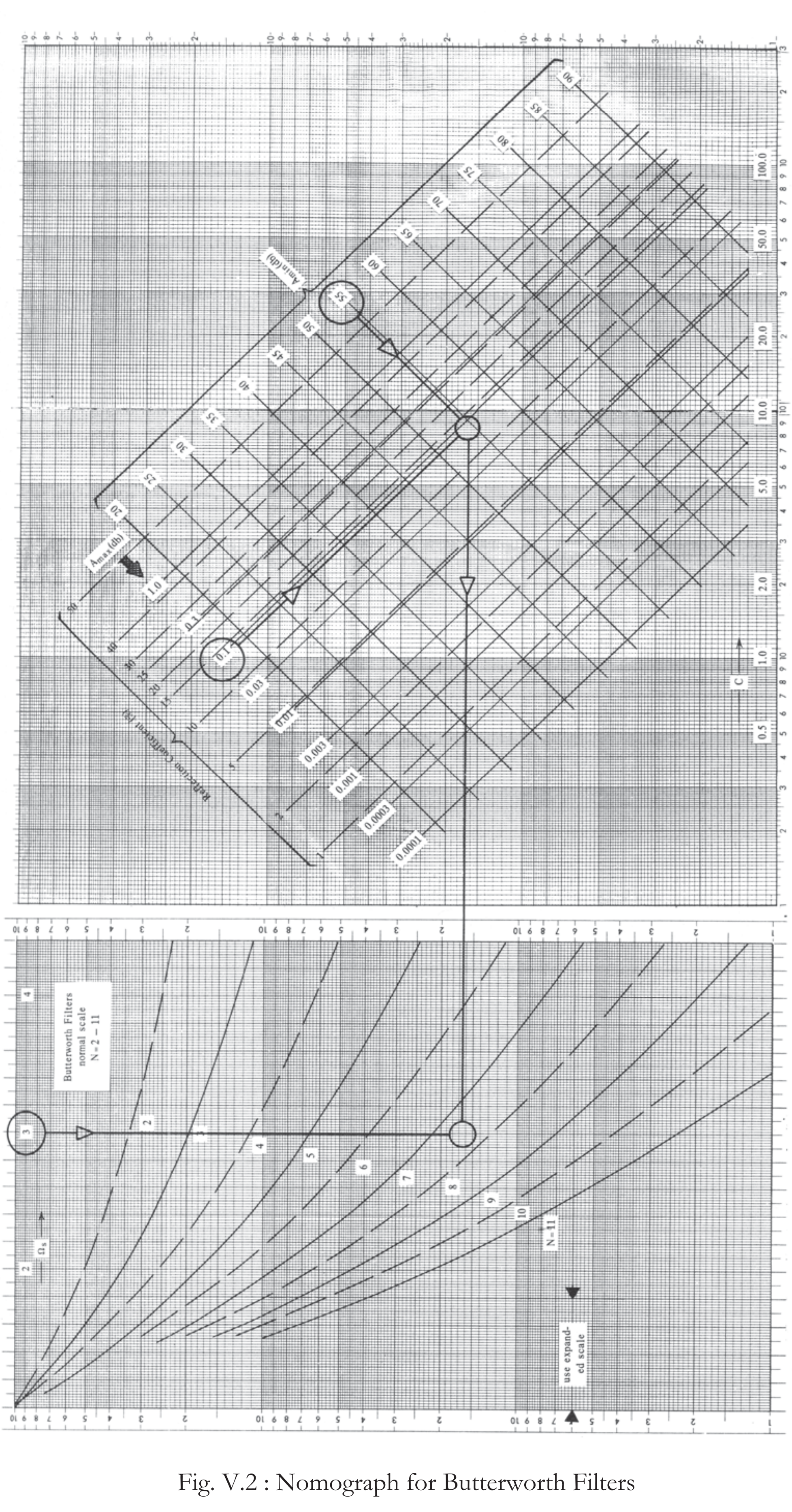


Fig. V.2 : Nomograph for Butterworth Filters

Calculation of the necessary degree

$A[dB] \geq A_{min}$ at $s = j\Omega_s$ yields

$$A_{min} \geq \log\,[1 + C^2\,\Omega_s^{2N}]$$

$$N \geq \frac{\log\,(L^{-2})}{\log\,\Omega_s}\;;\;\text{where } L^{-2} = \sqrt{\frac{10^{0.1\,A_{min}} - 1}{10^{0.1\,A_{max}} - 1}} \qquad (V.4)$$

Nomograph for the selection of the degree (Fig. V.2)

Normalize the frequencies with respect to the passband limit. In the nomograph on the right hand side, intersect the parameter lines "A_{max}" and "A_{min}" from the intersection, draw a horizontal line to the left.

Intersect this line with a vertical line through "Ω_S", the normalized stopband limit. The point of intersection falls between two parameter curves "N". Select the larger one.

Example: Design a lowpass with Butterworth response for the following specifications:

Passband limit f_p = 10 kHz.	A_{max} = 0.1 dB	
Stopband limit f_s = 30 kHz.	A_{min} = 55 dB	these are the
reference freq. = 10 kHz.		design parameters
cut-off rate = $f_s/f_p = \Omega_S = 3.0$		

Following the outlined procedure in the graph yields 7<N<8. It will therefore be necessary to use 8th degree. This leaves some margin for either the passband or the stopband or both.

Hurwitz polynomial

Denoting $F(s) = s^N$ and $P(s) = C^{-1}$ yields for the compatibility equation

$$F(s)\,F(-s) + P(s)\,P(-s) = \begin{cases} s^{2N} + C^{-2} = 0 & \text{if } N = \text{even} \\ s^{2N} - C^{-2} = 0 & \text{if } N = \text{odd} \end{cases} \qquad (V.5)$$

The roots are located on a circle with radius R. Those of E(s) are ([SA-1]):

$$s_{ov} = -R\,[\sin\,(\frac{2v-1}{N} \times \frac{\pi}{2}) \mp j\,\cos\,(\frac{2v-1}{N} \times \frac{\pi}{2})]$$

$$v = 1, 2, \ldots, [\frac{N}{2}] \qquad (V.6)$$

$R = \sqrt[N]{C^{-4}}$ in both cases.

Data Cards for SYNTH1 :

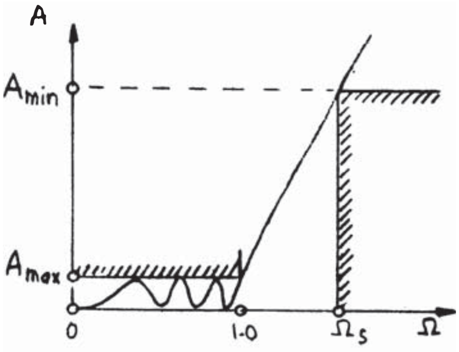
```
COLUMN   1    1    2    2    3    3    4    4    5    5    6    6    7    7
........0....5....0....5....0....5....0....5....0....5....0....5....0....5...
.
 APPROXIMATION ONLY                                                  K(S)-DESIG
 BUTTERWORTH RESPONSE LOWPASS                                         0 REAL
 DEGREE=  8  REFL.ZEROES      8 AT THE ORIGIN     0 PAIRS             0 QUADS
             ATTEN.POLES      0 AT THE ORIGIN     3 PAIRS
 REF.FRQ. 10.0    KHZ,  A0=   55.0     DB  AT     30.0     KHZ
 EVAL.   LOSS RESPONSE       PHASE RESPONSE      DELAY RESPONSE       STEP RESP.
 FROM      10.0    KHZ        0.0      KHZ        0.0      KHZ         0.010   M
 TO        50.0    KHZ        10.0     KHZ        10.0     KHZ         1.0     M
 SCALE   LINEAR              LINEAR              LINEAR               LOGARITHM.
 WITH      1.0     KHZ INCR.  0.1      KHZ INCR.  0.1      KHZ INCR.   40.0    F/
 PLOT MARGINS
 LEFT      0.0     DB         0.0      DB         0.0      DB          -0.2
 RIGHT     100.0   DB         810.0    DGR.       10.0     MSEC        1.8
 SUBDIV.   20.0    DB         90.0     DGR.       2.0      MSEC        0.2
 ALL REFL.ZEROES AT THE ORIGIN
 ALL ATTEN.POLES AT INFINITY
.
COLUMN   1    1    2    2    3    3    4    4    5    5    6    6    7    7
........0....5....0....5....0....5....0....5....0....5....0....5....0....5...
```

(b) Chebyshev and Invertid-Chebyshev Response

Both response types are closely related to Chebushev Polynomials $T_N(x)$ and their double inversion $T[(x^{-1})]^{-1}$.For real arguments and the particular value N=5, their behaviour is shown in Fig. V.3. Substituting (s/j) for x, both functions will display this performance along the imaginary axis. In this form they are of interest to synthesis.

Because of the close relationship between the two functions, the design equations are also closely related. To emphasize this fact, they will be displayed side by side.

Chebyshev Response



Significant: all reflections zeroes **distributed** over the passband range all attenuation poles at infinity.

Characteristic function

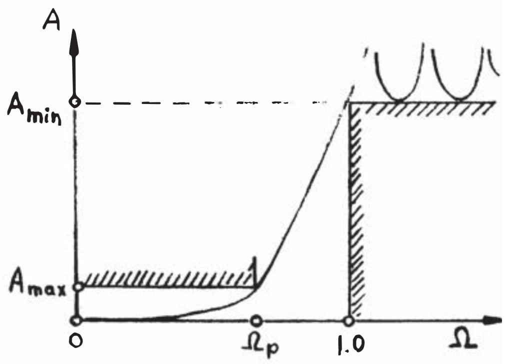
Chebyshev response

$$K_o(s) = T(s) \begin{cases} = \cos[N\cos^{-1}(\frac{s}{j})]; & |s| \leq 1 \\ = \cosh[N\cosh^{-1}(\frac{s}{j})]; & |s| > 1 \end{cases}$$

The magnitude of $K_o(s)$ has an upper boundary equal to 1 in the passband range

$$-j \leq s = j\Omega \leq +j$$

Inverted Chebyshev Response



Significant: **all attenuation** poles distributed over **the stopband range**; all reflection zeroes at the origin.

Inverted Chebyshev response

$$K_o(s) = \frac{1}{T(\frac{j}{s})} \begin{cases} = \dfrac{1}{\cos[N\cos^{-1}(\frac{j}{s})]}; & |s| > 1 \\ = \dfrac{1}{\cosh[N\cosh^{-1}(\frac{j}{s})]}; & |s| \leq 1 \end{cases} \quad \text{(V.7)}$$

The magnitude of $K_o(s)$ has a lower boundary equal to 1 in the stopband range

$$j < s = j|\Omega| < j\infty \quad \text{(V.8)}$$

In order to modify these boundaries a constant multiplier C_k may be added:

$$K(s) = C_k\, K_o(s) \quad \text{(V.9)}$$

The transfer loss is then:

$$A[dB] = 10 \log_{10} [1 + C_k^2\, K_o(s)\, K_o(-s)]_{s=j\Omega} \quad \text{(V.10)}$$

For a specified upper boundary A_{max} [dB] in the passband range the constant multiplier must have the value

$$C_k = 10^{0.1\, A_{max}} - 1$$

For a specified lower boundary A_{min} [dB] in the stopband range the constant multiplier must have the value

$$C_k = 10^{0.1\, A_{min}} - 1 \quad \text{(V.11)}$$

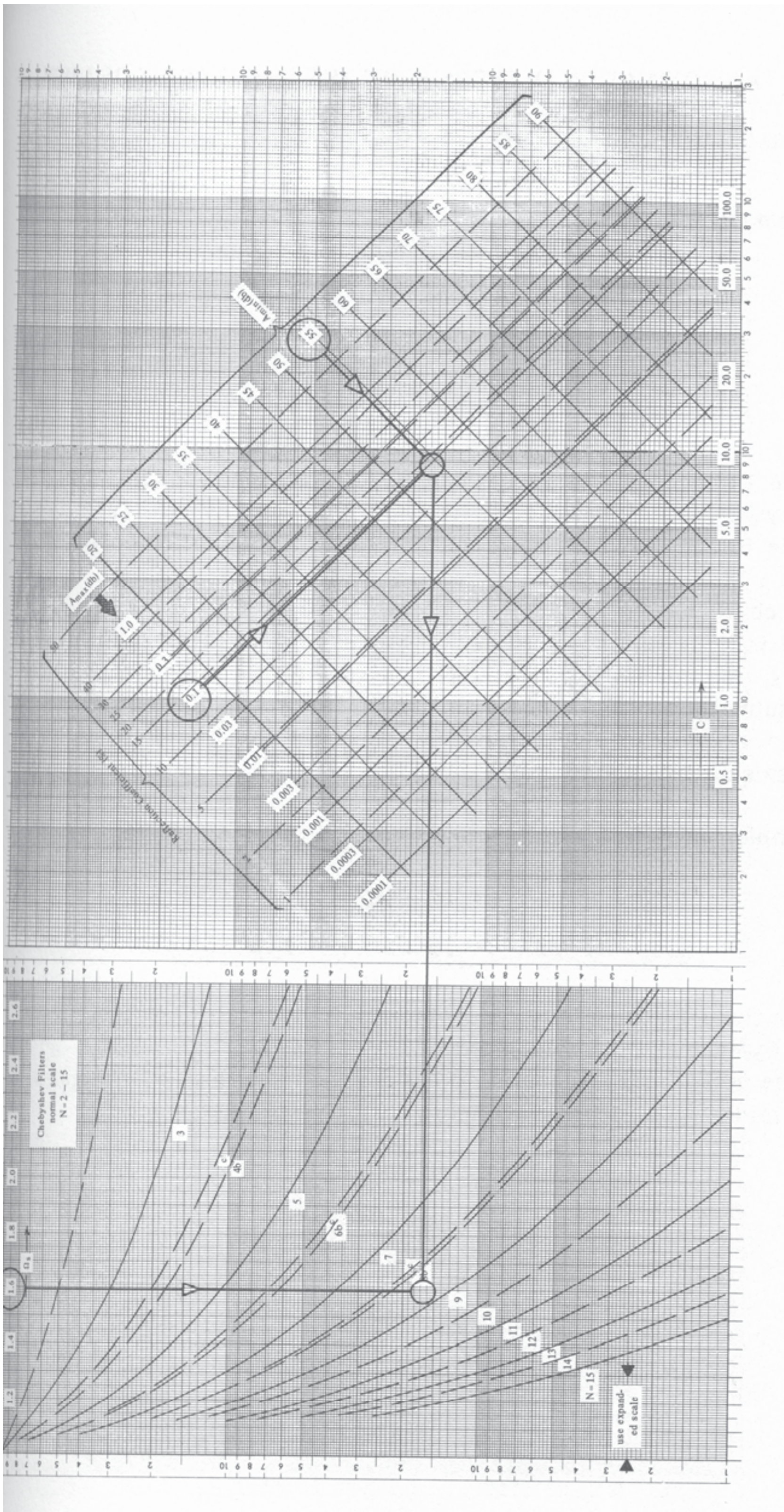


Fig. V.4a : Nomograph for Chebyshev Filters

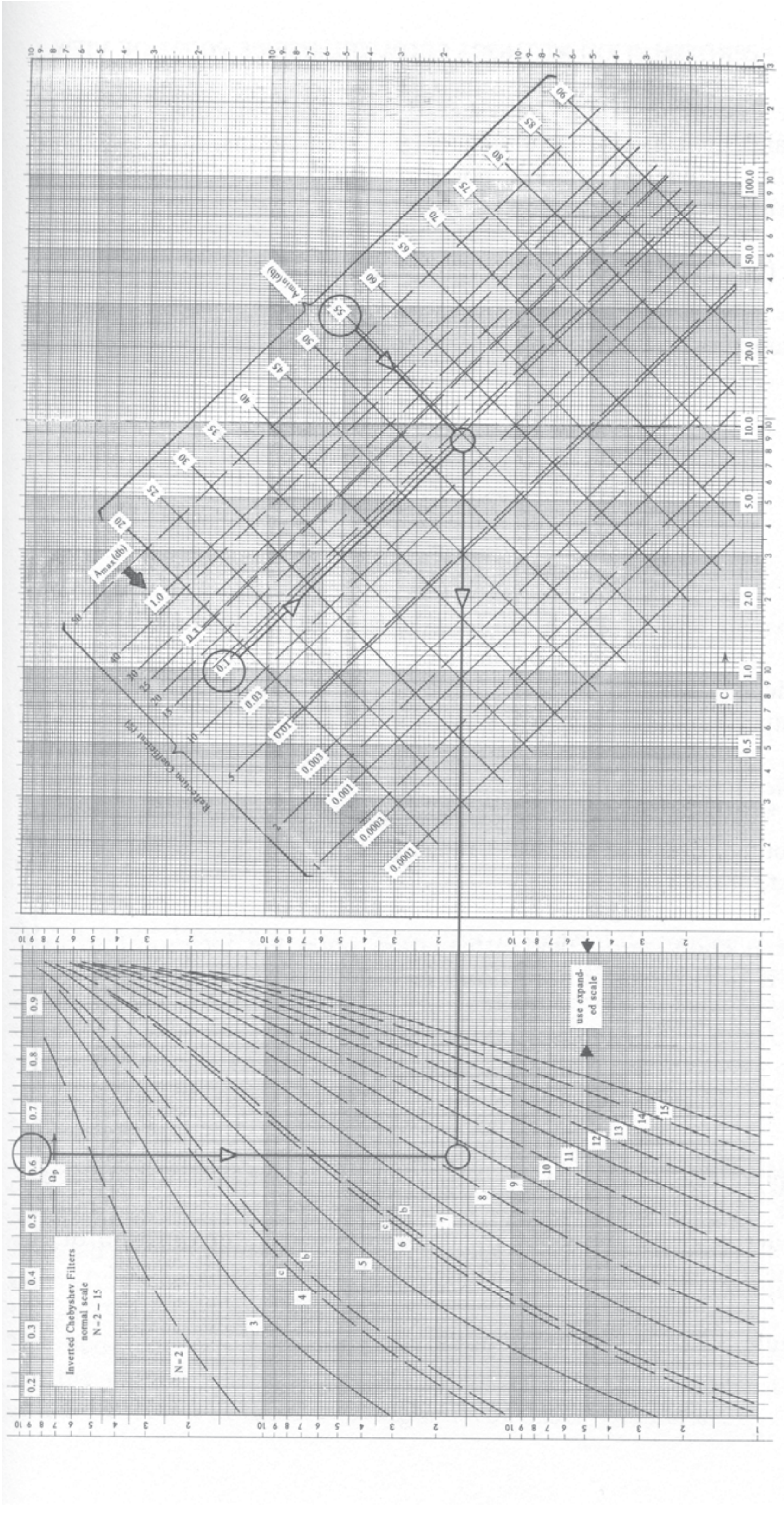


Fig. V.4b : Nomograph for Inverted Chebyshev Filters

For increasing values of $|\Omega|$ ($|\Omega|>1$), the transducer loss increases monotonically. It will reach a specified stopband loss A_{min} [dB] at the normalized stopband edge

$$s = j\Omega_S$$

Therefore from equations (7) and (11)

$$A_{min}[dB]=10\ \log_{10}[1+C_k^2 K_o(s)K_o(-s)]_{j\Omega_S}$$

For decreasing values of $|\Omega|$ ($|\Omega|<1$), the transducer loss decreases monotonically. It reaches a specified passband loss A_{max} [dB] at the normalized passband edge

$$s = j\Omega_P \tag{V.12}$$

Therefore from equations (7) and (11)

$$A_{max}[dB]=10\ \log[1+C_k K_o(s)K_o(-s)]_{j\Omega_P}$$

$$\Omega_S = \cosh\left[\frac{1}{N}\cosh^{-1}\frac{10^{0.1\ A_{min}}-1}{10^{0.1 A_{max}}-1}\right] = \frac{1}{\Omega_P} \tag{V.13}$$

Therefore,

$$N \geq \frac{\cosh^{-1}(L^{-2})}{\cosh^{-1}(\Omega_S)} = \frac{\cosh^{-1}(L^{-2})}{\cosh^{-1}(1/\Omega_P)} \tag{V.14}$$

with (L^{-2}) according to equation (4).

Equation (14) relates the degree N to the design parameters A_{max}, A_{min}, and Ω_S or Ω_P. For a specified set of these both response types will require the same degree. The choice between the two is made solely on practical considerations. The nomograph of Fig. V.4 is an aid to selecting the necessary degree. It is used in an analogous manner to the one for Butterworth response in Fig. V.2.

Reflection Zeroes of Chebyshev filters

Rather than to express the functions $T_N(s/j)$ by trigonometric functions it is often more convenient to employ root factors.

$$T_N(s/j) = 2^{N-1}\cdot s\cdot \prod^{n}\left(s^2+a_{2v}^2\right) \quad \text{for } N = 2n+1, \text{ odd}$$

(V.15)

$$T_N(s/j) = 2^{N-1}\cdot s\cdot \prod^{n}\left(s^2+b_{2v-1}^2\right) \quad \text{for } N = 2n, \text{ even}$$

$$a_{2v} = \sin\left(\frac{2v}{N}\right)\cdot\frac{\pi}{2}\ ;\quad |v| = 0,1,\ldots n$$

$$b_{2v-1} = \sin\left(\frac{2v-1}{N}\right)\cdot\frac{\pi}{2}\ ;\quad |v| = 0,1,\ldots n$$

Simultaneously, the a's and b's are also the reciprocals of the attenuation poles for the inverted Chebyshev response type, Therefore,

Characteristic functions

symmetrical case: $N = 2n+1$, odd

(V.16a)

$$K(s) = \frac{F(s)}{P(s)} = s^{N-1} \; C_k s \prod_{1}^{n} (s^2 + a_{2v}^2)$$

$$F(s) = s \prod_{1}^{n} (s^2 + a_{2v}^2) \; ; \; P(s) = [s^{N-1} \; C_k]^{-1}$$

(V.16b)

$$K(s) = \frac{C_k}{2^{N-1}} \; s \prod_{1}^{n} \frac{s^2}{(1 + s^2 a_{2v}^2)}$$

$$F(s) = \frac{C_k}{2^{N-1}} \; s^N \; ; \; P(s) = \prod_{1}^{n} (1 + s^2 a_{2v}^2)$$

antimetrical case: $N = 2n$, even

(V.17a)

$$K(s) = \frac{F(s)}{P(s)} = 2^{N-1} \; C_k \prod_{1}^{n} (s^2 + b_{2v-1}^2)$$

$$F(s) = \prod_{1}^{n} (s^2 + b_{2v-1}^2) \; ; \; P(s) = [2^{N-1} \; C_k]^{-1}$$

(V.17b)

$$K(s) = \frac{C_k}{2^{N-1}} \prod_{1}^{n} \frac{s^2}{(1 + s^2 b_{2v-1}^2)}$$

$$F(s) = \frac{C_k s^N}{2^{N-1}} \; ; \; P(s) = \prod_{1}^{n} (1 + s^2 b_{2v-1}^2)$$

Example: Design the characteristic functions of lowpass filters with Chebyshev and inverted Chebyshev response for the following specifications:

Passband limit $f_p = 10.0$ kHz $\quad A_{max} = 0.1$ dB

Stopband limit $f_s = 16.0$ kHz $\quad A_{min} = 55$ dB

(a) Chebyshev response

$f_{ref} = f_p = 10$ kHz;

cut-off rate $\Omega_s = f_s / f_p = 1.6$

From the nomograph Fig. V.4 $N = 9$

reflection zeroes, either from the equations (15) or from tables [CE-1]

(b) Inverted Chebyshev response

$f_{ref} = f_s = 16$ kHz

cut-off rate $\Omega_s = f_p / f_s = 0.625$

From the nomograph Fig. V.4. $N = 9$

$$\Omega_0 = 0.0$$
$$\Omega_{01} = 0.342020$$
$$\Omega_{02} = 0.642788$$
$$\Omega_{03} = 0.866025$$
$$\Omega_{04} = 0.984808$$
$$K(s) = C_k s \cdot (s^2 + \Omega_{01}^2)(s^2 + \Omega_{02}^2)(s^2 + \Omega_{03}^2)(s^2 + \Omega_{04}^2)$$

$$\Omega_{\infty 0} = \infty$$
$$\Omega_{\infty 1} = \Omega_{01}^{-1} = 2.92380$$
$$\Omega_{\infty 2} = \Omega_{02}^{-1} = 1.55572$$
$$\Omega_{\infty 3} = \Omega_{03}^{-1} = 1.15470$$
$$\Omega_{\infty 4} = \Omega_{04}^{-1} = 1.01542$$
$$K(s) = \frac{C_k \cdot s^9}{(s^2 + \Omega_{\infty 1}^2)(s^2 + \Omega_{\infty 2}^2)(s^2 + \Omega_{\infty 3}^2)(s^2 + \Omega_{\infty 4}^2)}$$

Data Cards for SYNTH1

```
COLUMN    1    1    2    2    3    3    4    4    5    5    6    6    7    7
.........0....5....0....5....0....5....0....5....0....5....0....5....0....5..
.
 APPROXIMATION ONLY
 CHEBYSHEV RESPONSE LOWPASS                                          K(S)-DES
 DEGREE=  9  REFL.ZEROES      1 AT THE ORIGIN      4 PAIRS             0 REAL
             ATTEN.POLES      0 AT THE ORIGIN      0 PAIRS             0 QUADS
 REF.FRQ. 10.0     KHZ,   A0=  55.0     DB  AT      16.0     KHZ
 EVAL.   LOSS RESPONSE        PHASE RESPONSE       DELAY RESPONSE      STEP RES
 FROM     10.0     KHZ         0.0      KHZ         0.0      KHZ        0.010
 TO       50.0     KHZ         10.0     KHZ         10.0     KHZ        1.0
 SCALE   LINEAR               LINEAR               LINEAR              LOGARITH
 WITH     1.0      KHZ INCR.   0.1      KHZ INCR.   0.1      KHZ INCR.  40.0
 PLOT MARGINS
 LEFT     0.0      DB          0.0      DB          0.0      DB         0.0
 RIGHT    100.0    DB          900.0    DGR.        1.0      MSEC       1.2
 SUBDIV.  20.0     DB          90.0     DEGR.       0.2      MSEC       0.4
 REFL.ZER.   RE(KHZ)              IM(KHZ)
 A(1)=    0.0          B(1)=   3.42020
 A(2)=    0.0          B(2)=   6.42788
 A(3)=    0.0          B(3)=   8.66025
 A(4)=    0.0          B(4)=   9.84808
 ALL ATTEN.POLES AT INFINITY
.
COLUMN    1    1    2    2    3    3    4    4    5    5    6    6    7    7
.........0....5....0....5....0....5....0....5....0....5....0....5....0....5..
```

```
COLUMN   1    1    2    2    3    3    4    4    5    5    6    6    7    7    8
.........0....5....0....5....0....5....0....5....0....5....0....5....0....5....0
.
 APPROXIMATION ONLY
 INVERTED CHEBYSHEV REPSONSE LOWPASS                                 K(S)-DESIGN
 DEGREE    9  REFL.ZEROES         9 AT THE ORIGIN      0 PAIRS        0 REAL
                ATTEN.POLES       0 AT THE ORIGIN      4 PAIRS        0 QUADS
 REF.FRQ. 16.0      KHZ,     A0=  55.0    DB  AT       16.0     KHZ
 EVAL.    LOSS RESPONSE          PHASE RESPONSE        DELAY RESPONSE       STEP RESP.
 FROM       10.0      KHZ         0.0       KHZ         0.0       KHZ        0.010    MS
 TO         20.0      KHZ        16.0       KHZ        16.0       KHZ        1.0      MS
 SCALE    LINEAR                 LINEAR                LINEAR               LOGARITHM.
 WITH       1.0       KHZ INCR.   0.1       KHZ INCR.   0.1       KHZ INCR.  40.0     F/D
 PLOT MARGINS
 LEFT       0.0       DB          0.0       DB          0.0       DB         0.0      V
 RIGHT      80.0      DB        900.0       DGR.        1.0       MSEC       1.2      V
 SUBDIV.    20.0      DB         90.0       DEGR.       0.2       MSEC       0.2      V
 ALL REFL.ZEROES AT THE ORIGIN
 ATTEN.PLS.   RE(KHZ)                   IM(KHZ)
 X(1)=      0.0            Y(1)=  46.780
 X(2)=      0.0            Y(2)=  24.891
 X(3)=      0.0            Y(3)=  18.475
 X(4)=      0.0            Y(4)=  16.247

COLUMN   1    1    2    2    3    3    4    4    5    5    6    6    7    7    8
.........0....5....0....5....0....5....0....5....0....5....0....5....0....5....0
```

The Roots of the Hurwitz polynomial E(s) are found in the conventional way. For the Chebyshev response type, they may be expressed in explicit form [SA-1]

$$s_v = \underbrace{a_e \sin\left[\frac{2v-1}{N}\frac{\pi}{2}\right]}_{-a_v} \pm j \underbrace{\sqrt{1+a_e^2}\cos\left[\frac{2v-1}{N}\frac{\pi}{2}\right]}_{b_v} \tag{V.18}$$

$$v = 1, 2, \ldots \left[\frac{N}{2}\right]$$

with

$$a_e = \sqrt{10^{-0.1\,A_{max}} - 1} \tag{V.19}$$

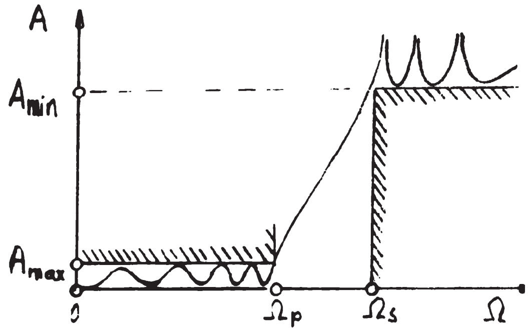
For the inverted Chebyshev type, the roots of E(s) are reciprocal to those belonging to an auxiliary Chebyshev lowpass of the same degree. The design parameters of this auxiliary lowpass, A_{max} and Ω_s, are

$$A_{max}[\mathrm{dB}] = 10 \log_{10} [1 + (10^{0.1A_{min}} - 1)^{-1}] \quad ; \quad \Omega_s = \Omega_p^{-1} \qquad \text{(V.20)}$$

where A_{min} and Ω_p are the design parameters of the desired lowpass. For inverted Chebyshev filters, a table of normalized elements can be found in the Appendix. This table is arranged in an identical form as [SA-2]. For denormalization, note that $f_{ref} = f_s$, i.e. the stopband limit.

(c) Cauer Parameter response

Significant for this type:
All reflections zeroes distributed over the passband range;
all attenuation poles distributed over the stopband range.
References: [CA-1], pgs. 446-459; [GU-1] pgs. 607-614; [WE-1], pgs. 532-533.



Characteristic function

$$K_0(s) = \frac{s\prod_1^n (s^2 + a_{2v}^2)}{\prod_1^n (s^2 a_{2v}^2 + 1)} \qquad N = 2n + 1, \text{ odd}$$

$$K_o(s) = \frac{\prod_1^n (s^2 + a_{2v-1}^2)}{\prod_1^n (s^2 a_{2v-1}^2 + 1)} \qquad N = 2n, \text{ even} \qquad \text{(V.21)}$$

Note that for both functions

$$K_o\left(\frac{1}{s}\right) = \frac{1}{K_o(s)} \qquad \text{(V.22)}$$

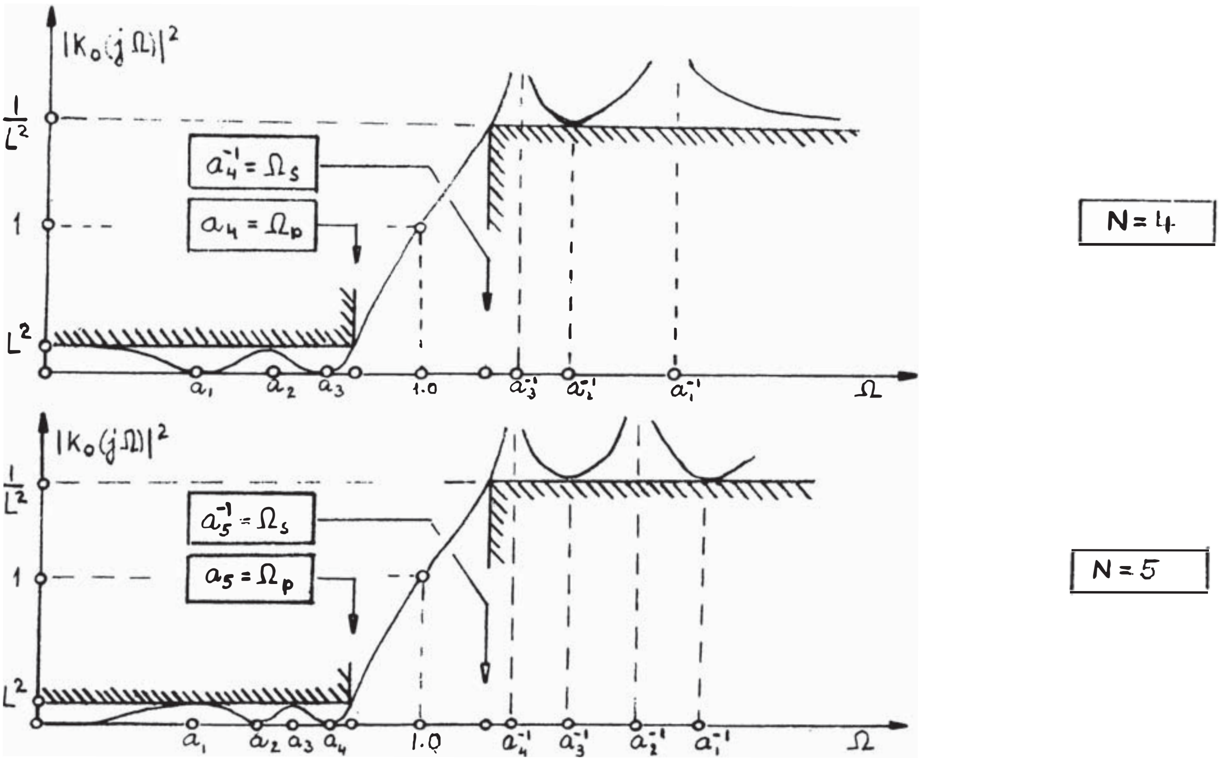


Figure V. 5 Cauer Response for 4th or 5th Degree Functions

regardless of parameter values. Consequently,

$$0 \le |K_o(j\Omega)| \le L \quad \rightarrow \quad \frac{1}{L} \le |K_o(j\Omega)| < \infty$$
$$0 \le \Omega \le \Omega_p < 1 \quad \leftarrow \quad 1 < \Omega_s = \Omega_p^{-1} < \infty \qquad (V.23)$$

By a proper selection of the parameters a_i one can achieve

$$|K_o(j\Omega)| = L \qquad 0 \le \Omega \le \Omega_p < 1 \qquad (V.24)$$

at all maxima and also the edge of the passband. Then because of equation (22) all minima and also the edge of the stopband will satisfy

$$|K_o(j\Omega)| = \frac{1}{L} \qquad \Omega_s \le \Omega < \infty \qquad (V.25)$$

This is obviously the optimum which can be achieved. The parameters for this case are elliptic functions of the degree and the normalized passband limit.

$$a_v = \sqrt{\sin\Theta}\ \mathrm{sn}\left(\frac{v}{N}K;\ \Theta\right);\ v = 1, 2, \ldots N$$
$$a_N = \sqrt{\sin\Theta} = \Omega_p \text{ (passband limit)} \qquad (V.26)$$

The quantity "K" is the complete elliptic integral of first kind

$$K = \int_0^1 \frac{dx}{\sqrt{(1 - x^2)(1 - x^2 \sin^2 \Theta)}} \qquad (V.27)$$

Furthermore,

$$L = \frac{1}{a_N} \prod_1^{n+i} a_{2v-1}^2 \quad \text{for } N = 2n+1 \text{, odd} \qquad (V.28)$$
$$L = \prod_1^{n} a_{2v-1}^2 \quad \text{for } N = 2n \text{, even}$$

Therefore for a specified degree N of the characteristic function the parameter a_i as well as the quantity L depend only on $\Omega = \sqrt{\sin\Theta}$, which is called the modular angle. In honor to Cauer who introduced them first to filter design, they are called "Cauer parameters". They have been published with 6 significant digits and up to 12[th] degree by Glowatzki([GL-1]). Numerical calculation of these parameters can be carried out by a method published by Orchard [OR-1].

The addition of a factor C_k in equation (21) causes the following changes in the subsequent equations:

(V.21) $$K(s) = C_k K_o(s) \tag{V.29}$$

(V.24)
$$\begin{aligned} &0 \le |K(j\Omega)| \le C_k L && \text{in the passband range} \\ &\frac{C_k}{L} \le |K(j\Omega)| < \infty && \text{in the stopband range} \end{aligned} \tag{V.30}$$

Consequently,

$$\begin{aligned} &A_{max}[\mathrm{dB}] = 10 \log_{10}(1 + C_k^2 L^2) && \text{in the passband range} \\ &A_{min}[\mathrm{dB}] = 10 \log_{10}(1 + \frac{C_k^2}{L^2}) && \text{in the stopband range} \end{aligned} \tag{V.31}$$

and

$$\begin{aligned} C_k &= \sqrt{(10^{0.1 A_{max}} - 1)(10^{0.1 A_{min}} - 1)} \\ L &= \sqrt{(10^{0.1 A_{max}} - 1)/(10^{0.1 A_{min}} - 1)} \end{aligned} \tag{V.32}$$

In equations (32), the right hand sides depend only on the specifications; the quantity L depends on the degree and the cut-off rate. As previously with the other response types, the requirements are most easily matched to a necessary degree by means of the nomograph in Fig. V.6.

Example: Design the characteristic function of a lowpass filter with Cauer parameter response for the following specifications:

Passband limit $f_p = 10$ kHz $A_{max} = 0.1$ dB

Stopband limit $f_s = 15$ kHz $A_{min} = 55$ dB

reference frequency $f_{ref} = \sqrt{f_p f_s} = 12.25$ kHz

nominal passband lim. $\Omega_p = f_p/f_{ref} = 0.815$

modular angle $\Theta = \arcsin(\Omega_p^2) \sim 42°$

From the design graph (Fig. V.6) one obtains N = 6 and $C_k = 0.94$

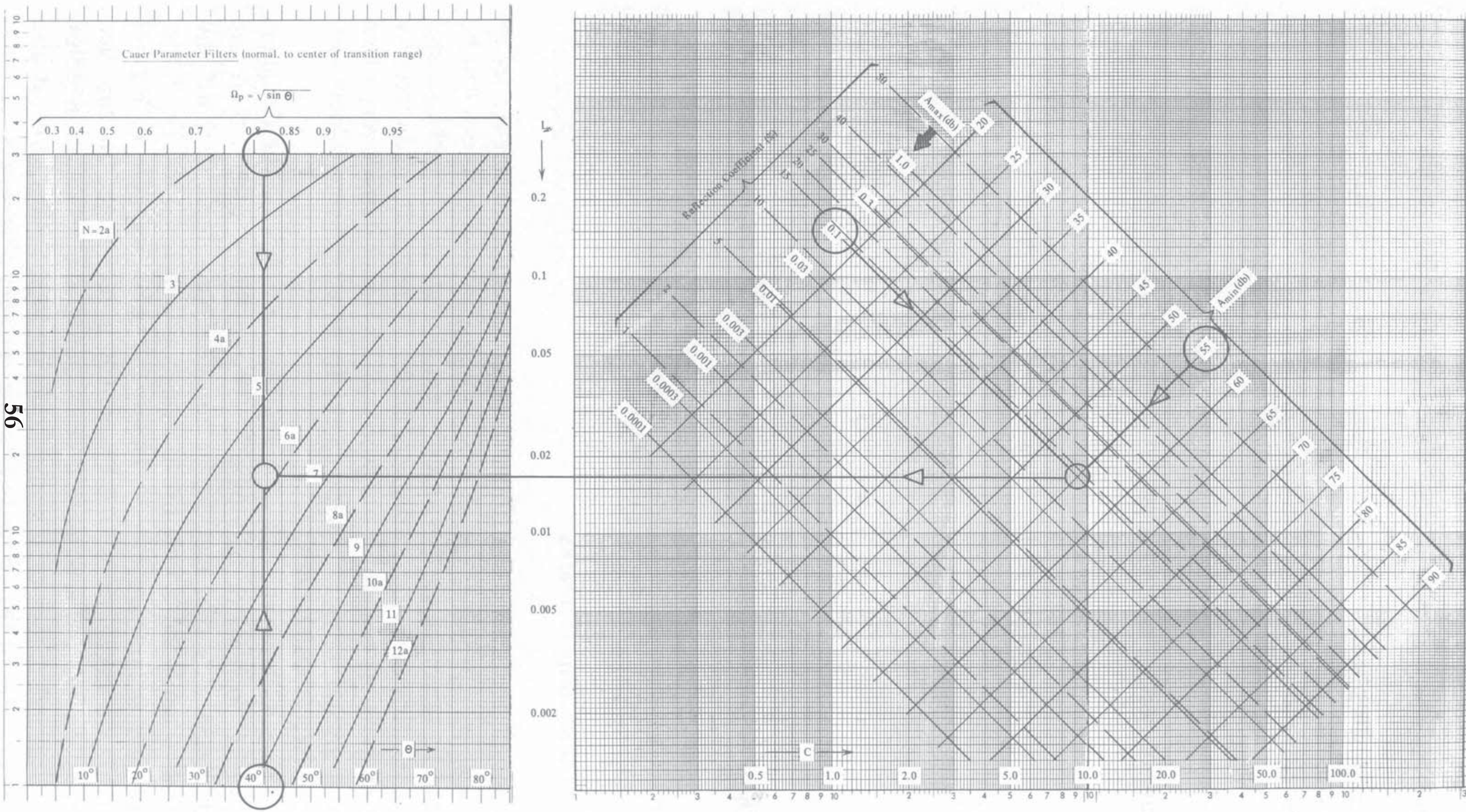


FIGURE V : Nomograph for Cauer filters.

For N = 6 and $\Theta = 42°$ one finds in the Glowatzki tables the following Cauer parameters:

$a_1 = 0.241\ 746$ $\quad a_3 = 0.619\ 568$ $\quad a_5 = 0.797\ 209$

$a_2 = 0.454\ 326$ $\quad a_4 = 0.732\ 713$ $\quad a_6 = 0.818\ 004$

For the significance of the parameters see Fig. V. With these parameters, the characteristic function becomes

$$K(s) = 0.94\ \frac{(s^2 + 0.241746^2)(s^2 + 0.619568^2)(s^2 + 0.797209^2)}{(0.241746^2 s^2 + 1)(0.619568^2 s^2 + 1)(0.797209^2 s^2 + 1)}$$

For several reasons, it is more practical to normalize the Cauer parameters with respect to the passband limits. (In equations (26), leave out the factor $\sqrt{\sin \Theta}$.) In this form, they are tabulated in [CE-1]. The input data for SYNTH1 are based on these parameters.

Input data for SYNTH1

```
COLUMN   1    1    2    2    3    3    4    4    5    5    6    6    7    7    8
.........0....5....0....5....0....5....0....5....0....5....0....5....0....5....0
.
 COMPLETE SYNTHESIS
 CAUER PARAMETER LOWPASS N=6,THETA=42 DEG.                          K(S)-DESIGN
 DEGREE=  6   REFL.ZEROES        0 AT THE ORIGIN      3 PAIRS        0 REAL
              ATTEN.POLES        0 AT THE ORIGIN      3 PAIRS        0 QUADS
 REF.FRQ. 10.0      KHZ,    A0=  55.0     DB  AT      14.9488 KHZ
 EVAL.   LOSS RESPONSE          PHASE RESPONSE       DELAY RESPONSE      STEP RESP.
 FROM       10.0     KHZ         0.0      KHZ         0.0      KHZ        0.010    MS
 TO         20.0     KHZ        10.0      KHZ        10.0      KHZ        1.0      MS
 SCALE     LINEAR               LINEAR               LINEAR              LOGARITHM.
 WITH       0.2      KHZ INCR.   0.2      KHZ INCR.   0.2      KHZ INCR.  40.0     F/D
 PLOT MARGINS
 LEFT       0.0      DB          0.0      DB          0.0      DB        -0.5      V
 RIGHT     80.00     DB        630.0      DGR.       10.0      MSEC       1.5      V
 SUBDIV.   20.0      DB         90.0      DGR.        1.0      MSEC       0.25     V
 REFL.ZEROES RE(KHZ)                IM(KHZ)
 A(1)=      0.0           B(1)=  2.95532
 A(2)=      0.0           B(2)=  7.57414
 A(3)=      0.0           B(2)=  7. B(3)=   9.74578
 ATTEN.POLES RE(KHZ)                IM(KHZ)
 X(1)=      0.0           Y(1)=  15.334597446
 X(2)=      0.0           Y(2)=  19.734295186
 X(3)=      0.0           Y(3)=  50.569105987
 REALIZATION DATA
 GEN.              150.0    OHM
   1 REALIZATION
D  2   1   3   4
.
COLUMN   1    1    2    2    3    3    4    4    5    5    6    6    7    7    8
.........0....5....0....5....0....5....0....5....0....5....0....5....0....5....0
```

Hurwitz Polynomial

By means of elliptic functions, it is possible to write expressions for the natural modes similar to those of Chebyshev filters [CA-1]. These formulas used to be valuable aids to calculate the roots when these calculations had to be carried out by desk calculators. With computers, these formulas lost their importance.

2. Equal ripple passband response by means of Cauer's q-functions

In many practical applications, it is desirable to design transmission netwo with equal-ripple passbands and an arbitrary set of attenuation poles. A convenient means to solve problems of this type is Darlington's reference filter method in combination with Cauer's q-functions ([DA-1]; [CA-1], pages 458-468; [FE-1]). According to this concept, the characteristic function of the desired network is obtained from the functions $\cosh\Gamma$ and $\sinh\Gamma$ of suitable image parameter filters having a propagation constant Γ.

Historically the design of such filters is closely related to the theory of homogeneous transmission lines. According to this theory, the input and output quantities of currents and voltages of such a line segment are related by the expressions

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} \cosh\Gamma & Z_0 \sinh\Gamma \\ Z_0^{-1} \sinh\Gamma & \cosh\Gamma \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} \tag{V.33}$$

where Z_0, the characteristic impedance, and Γ, the propagation constant depend on the parameters of the transmission line. All elements of A_{ij} of the chain matrix of equation (V.33) are transcendental functions of the complex variable s.

Somewhat artificially, the concepts and notation of transmission lines have been carried over to symmetrical sections which are composed of lumped circuit elements, similar to those shown in Fig. V.7. It is customary to write the chain matrix of such sections in the same form as equation (V.33). Thus, the overall chain matrix becomes

$$(A) = \prod_i (A_i) = \prod_{(i)} \begin{pmatrix} \cosh\Gamma_i & Z_{oi} \sinh\Gamma_i \\ Z_{oi}^{-1} \sinh\Gamma_i & \cosh\Gamma_i \end{pmatrix} = \prod_{(2)} \begin{pmatrix} a_{11}^{(i)} & a_{12}^{(i)} \\ a_{21}^{(i)} & a_{22}^{(i)} \end{pmatrix} \tag{V.34}$$

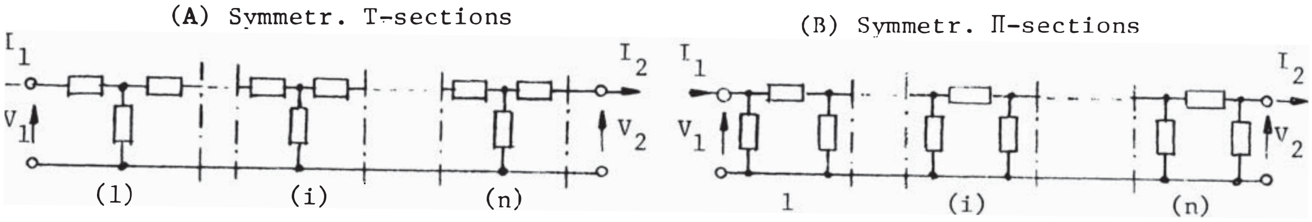
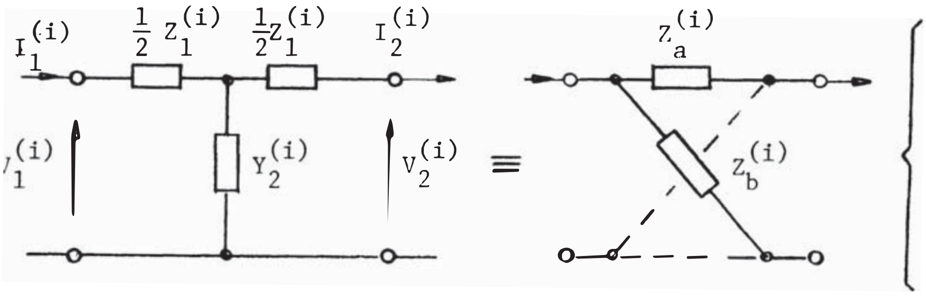
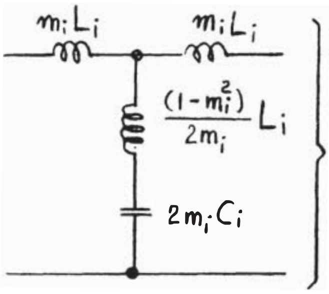


Figure V.7 Ladder Circuits Composed of Symmetrical Sections



$$\left\{\begin{aligned} Z_a^{(i)} &= \frac{1}{2} Z_1^{(i)} \\ Z_b^{(i)} &= \frac{1}{2} Z_1^{(i)} + 2 Z_2^{(i)} \end{aligned}\right.$$

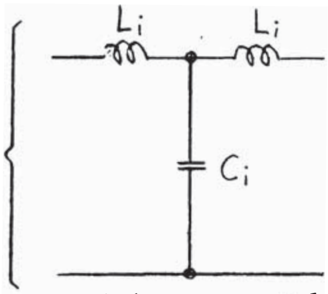
Figure V.8 A symmetrical T-Section and its Lattice Equivalent



From Fig. (V.8) and equation (V.40):

$$\boxed{q_i(s) = \frac{m_i s}{\sqrt{s^2+1}}}$$

$[f_{ref} = (2\pi \sqrt{L_i C_1})^{-1}]$



special case $m_i = 1$

Figure V.9 Elementary q-Functions

However, because now the sections consist of lumped elements, the matrix elements will be rational functions of s.

Neither in Fig. V.7 nor in equation (V.34) is it implied that some or all cascaded sections are identical. However, all sections may have been designed for a common characteristic impedance Z_o, thus

$$Z_{o1} = Z_{o2} = \ldots Z_{oi} = \quad Z_{on} = Z_o \tag{V.35}$$

in which case the matrix product of equation (V.34) yields

$$(A) = \prod_{(i)} \begin{matrix} \cosh\Gamma_i & Z_o \sinh\Gamma_i \\ Z_o^{-1}\sinh\Gamma_i & \cosh\Gamma_i \end{matrix} = \begin{matrix} \cosh\Gamma & Z_o \sinh\Gamma \\ Z_o^{-1}\sinh\Gamma & \cosh\Gamma \end{matrix} \tag{V.36}$$

with $$\Gamma = \Gamma_1 + \Gamma_2 + \ldots \Gamma_i + \ldots + \Gamma_n = \Sigma\Gamma_i \tag{V.37}$$

The various methods to achieve a common Z_o (which is a major objective of image parameter theory) have no significance for network synthesis. Of greater importance is the quantity $\cosh\Gamma$ which actually is a rational function with the same poles and zeroes as the desired characteristic function.

To derive the composite propagation function of equations (V.36) and (V.37), the symmetrical T section of Fig. V.8 is considered. For this section

$$\frac{V_1^{(i)}}{V_2^{(i)}} = A_{11}^{(i)} = \cosh\Gamma_i = 1 + \frac{1}{2} z_1^{(i)} y_2^{(i)} \tag{V.38}$$

or in terms of the equivalent lattice branches

$$\cosh\Gamma_i = \frac{1}{2}\left[e^{\Gamma_i} + e^{-\Gamma_i}\right] = \frac{z_a^{(i)} + z_b^{(i)}}{z_a^{(i)} - z_b^{(i)}} = \frac{q_i^2 + 1}{q_i^2 - 1} \tag{V.39}$$

with $$q_i^2 = \frac{z_a^{(i)}}{z_b^{(i)}} \; ; \; q_i = \sqrt{\frac{z_a^{(i)}}{z_b^{(i)}}} \tag{V.40}$$

From equation (V.39) one may derive

$$e^{\Gamma_i} = \frac{q_i + 1}{q_i - 1} = Q_i \tag{V.41}$$

where Γ_i is the propagation constant of one section. The composite propagation constant Γ as defined by equation (V.37) satisfies the following relation:

$$e^{\Gamma} = e^{\Sigma\Gamma_i} = \prod_{(i)} \frac{q_i + 1}{q_i - 1} = \prod_{(i)} Q_i \tag{V.42}$$

For sections containing only reactive elements, the quantity q_i of equation (V.40) belongs to a class of functions which Cauer has termed "q-functions". They possess the following properties:

(a) $q(s)$ is positive real;

(b) for s-values on the imaginary axis, $q(s)$ is either real or purely imaginary;

(c) the real and imaginary intervals on the j-axis are separated by branching points of the order "$\frac{1}{2}$". At any such branching point $s = s_1$, $q(s)$ assumes either of the following forms:

$$q(s) = \sqrt{s - s_1}\; r(s) \quad \text{or} \quad q(s) = \frac{r(s)}{\sqrt{s - s_1}} \tag{V.43}$$

with $r(s) \neq 0$ and regular;

the branching points are the simple zeroes of $z_a^{(i)}/z_b^{(i)}$.

(d) all zeroes and poles of q-functions must be in the imaginary intervals; in the real intervals, the q-functions are positive and finite.

Obviously,

in the imaginary interval of $q_i(s)$

$$\left|\frac{q_i + 1}{q_i - 1}\right| = 1; \; \Gamma_i = jb_i \tag{V.44}$$

in the real interval of $q_i(s)$

$$\left|\frac{q_i + 1}{q_i - 1}\right| \geq 1; \; \Gamma_i = a_i; \tag{V.45}$$

q-functions having identical real and imaginary intervals will be called "compatible". Such functions can be composed to form compatible q-functions of higher complexity. The composition follows the rule of equation (V.42)

$$e^{\Gamma} = \frac{q + 1}{q - 1} = \frac{q_1 + 1}{q_1 - 1} \; \frac{q_2 + 1}{q_2 - 1} \; \frac{q_3 + 1}{q_3 - 1} \cdots \frac{q_n + 1}{q_n - 1} \tag{V.46}$$

The product on the right hand side defines the composite q-functions on the left. For the composition of q-functions it is practical to use the elementary q-functions of Fig. V.9 as building blocks. Each of these may be designed to provide a pair of attenuation poles at a specified frequencies $\pm s_\infty$.

$$q_i(s_\infty) = 1 \quad \text{yields} \quad m_i = \frac{\sqrt{s_\infty^2 + 1}}{s_\infty} \tag{V.47}$$

The composite q-functions will assume the value 1 at all specified pole frequencies. Consequently, the related propagations constant becomes infinite. From the composite q=function one may form the function $K_o(s)$

$$K_o(s) = \cosh\Gamma = \frac{1}{2}\left[\frac{q+1}{q-1} + \frac{q-1}{q+1}\right] = \frac{q^2+1}{q^2-1} \tag{V.48}$$

which possess the following properties:

(a) in the imaginary ranges of q(s)

$\Gamma = jb$; $\cosh jb = \cos b$

$-1 \leq \cosh jb = \cos b \leq +1$

(b) in the real ranges of q(s)

$\Gamma = a \geq 0$, real ; $\cosh a \geq +1$

$\Gamma = \infty$ wherever one of the contributing q-functions assumes the value of 1.

These properties make $\cosh\Gamma$ the desired characteristic function except for an optional scaling constant C_k. Thus

$$K(s) = C_k\, K_o(s) = C_k \cosh\Gamma \tag{V.49}$$

Obviously, the elementary q-functions of Fig. V.9 have an imaginary interval

$$-j \leq s = \Omega \leq +j$$

and, therefore, the derived networks are lowpass filters.

Example: Design the characteristic function of a lowpass for an equal-ripple passband ranging from 0 to 12 kHz and specified attenuation poles at 24 kHz and 36 kHz. Passband ripple = 0.3 dB.

(a) Normalization of frequencies. $f_p = f_{ref} = 12$ kHz

$f_{\infty 1} = 24$ kHz $\qquad s_{\infty 1} = j\Omega_{\infty 1} = j\,2$

$f_{\infty 2} = 36$ kHz $\qquad s_{\infty 2} = j\Omega_{\infty 2} = j\,3$

(b) Elementary q-functions

$$q_1 = \frac{m_1 s}{\sqrt{s + 1}}\Bigg|_{s_{\infty 1}} = 1 \quad \rightarrow \quad m_1 = \sqrt{1 - \frac{1}{\Omega_{\infty 1}^2}} = \sqrt{0.750} = 0.866$$

$$q_2 = \frac{m_2 s}{\sqrt{s + 1}}\Bigg|_{s_{\infty 2}} = 1 \quad \rightarrow \quad m_2 = \sqrt{1 = \frac{1}{\Omega_{\infty 2}^2}} = \sqrt{0.888} = 0.942$$

(c) Composite q-functions

$$\frac{q + 1}{q - 1} = \frac{q_1 + 1}{q_1 - 1}\,\frac{q_1 + 1}{q_1 - 1} = \frac{m_1 s + \sqrt{s^2 + 1}}{m_1 s - \sqrt{s^2 + 1}}\,\frac{m_2 s + \sqrt{s^2 + 1}}{m_2 s - \sqrt{s^2 + 1}}$$

$$= \frac{[(m_1 m_2 + 1)s^2 + 1] + (m_1 + m_2)s\sqrt{s^2 + 1}}{[(m_1 m_2 + 1)s^2 + 1] - (m_1 + m_2)s\sqrt{s^2 + 1}}$$

thus $$q(s) = \frac{(m_1 m_2 + 1)\ s^2 + 1}{(m_1 + m_2)\ s\ \sqrt{s^2 + 1}} = \frac{1.816\ s^2 + 1}{1.808\ s\ \sqrt{s^2 + 1}}$$

(d) Characteristic function

$$K_o(s) = \frac{q^2 + 1}{q^2 - 1} = \frac{(1.8157\ s^2 + 1)^2 + (1.808\ s\ \sqrt{s^2 + 1})^2}{(1.8157\ s^2 + 1)^2 - (1.808\ s\ \sqrt{s^2 + 1})^2}$$

$$K_o(s) = 235.3\ \frac{s^4 + 1.0509\ s^2 + 0.523}{s(s^4 + 13\ s^2 + 36)}$$

For an upper boundary of $A_{max} = 0.3$ dB in the passband, the actual characteristic function becomes

$$K(s) = \sqrt{10^{0.03} - 1}\ K_o(s)$$

In the described form, the procedure always yields characteristic functions of even degree with pole pairs on either axis or pole quadruplets. In the lowpass case, the value 1 for one or several of the m_i's produces poles of even order at infinity.

An odd-degree pole at infinity requires the addition of a first order pole to any pole pairs which may already exist. In terms of Fig. V.7, such poles are related to half sections with half the propagation constant of a full section. Bisecting a full section and applying the composition rule yields

$$\frac{q_i + 1}{q_i - 1} = \frac{q'_{1/2} + 1}{q'_{1/2} - 1} \; \frac{q'_{1/2} + 1}{q'_{1/2} - 1} \tag{V.50}$$

$$q_i = \tfrac{1}{2}\left(q'_{1/2} + \frac{1}{q'_{1/2}}\right) \qquad q'_{1/2} = q_i \pm \sqrt{q_i^2 - 1} \tag{V.51}$$

where q_i is the elementary q-function of a full section, and $q'_{1/2}$, by definition, is the elementary q'-function of a half section.

In the lowpass case, substituting the expression of Fig.V.9 for q_i one obtains

$$q'_{1/2} = \frac{m_i s \pm \sqrt{(m^2 - 1)\, s^2 - 1}}{\sqrt{s^2 + 1}} \tag{V.52}$$

For composition with other q-functions this expression is meaningful only for the particular parameter value $m_i = 1$, i.e. for the bisection of a section which produces a pole at infinity. For this particular m_i,

$$q'_{1/2} = \sqrt{\frac{s + j}{s - j}} \tag{V.53}$$

or its reciprocal. This function is the elementary member of a class which Cauer has called q'-functions. They possess the same properties as q-functions except one, namely to be real for real values of the argument s. Instead, for real values of s, $|q'(s)| = 1$.

The propagation function $\Gamma'_{1/2}$ of a half section is defined by

$$e^{\Gamma'_{1/2}} = \pm j \, \frac{q'_{1/2} + 1}{q'_{1/2} - 1} \tag{V.54}$$

or

$$e^{(\Gamma'_{\frac{1}{2}} \pm j\frac{\Pi}{2})} = \frac{q'_{\frac{1}{2}} + 1}{q'_{\frac{1}{2}} - 1} \qquad (V.55)$$

Composite q'-functions are generated in similar manner as composite q-functions. In analogy to equation (V.42),

$$e^{\Gamma}\, e^{(\Gamma'_{\frac{1}{2}} \mp j\frac{\Pi}{2})} = [\,\Pi \frac{qi + 1}{qi - 1}\,] \quad \frac{q_{\frac{1}{2}} + 1}{q_{\frac{1}{2}} - 1} = \frac{q' + 1}{q' - 1} \qquad (V.56)$$

$$e^{(\Gamma' \mp j\frac{\Pi}{2})} = \frac{q' + 1}{q' - 1} \qquad (V.57)$$

Therefore,

$$\cosh(\Gamma' \mp j\frac{\Pi}{2}) = \begin{cases} = \cosh\Gamma' \cos \tfrac{1}{2}\Pi \mp j \sinh\Gamma' \sin \tfrac{1}{2}\Pi = \mp j \sinh\Gamma' \\[2ex] = \tfrac{1}{2}[\frac{q' + 1}{q' - 1} + \frac{q' - 1}{q' + 1}] = \frac{q'^2 + 1}{q'^2 - 1} \end{cases}$$

and by comparison,

$$\sinh\Gamma' = \pm j \frac{q'^2 + 1}{q'^2 - 1} \qquad (V.58)$$

Because equations (V.44) and (V.45) hold also for q'-functions one may write in analogy to equation (V.48) and (V.49)

$$K_o(s) = \sinh\Gamma' = \pm j \frac{q'^2 + 1}{q'^2 - 1} \qquad (V.60)$$

$$K(s) = C_k\, K_o(s) = C_k \sinh\Gamma' \qquad (V.61)$$

Example. Design the characteristic function of a lowpass of 3rd degree for an equal-ripple passband from 0 to 12 kHz and one specified attenuation pole at 24 kHz.

(a) Normalization as in the previous example

(b) Elementary q-function as q_1 in the previous example

$$q_1(s) = \frac{0.866\ s}{\sqrt{s^2 + 1}}$$

	Location of Atten. Poles	m_i	Range for m_i	Elementary q-function
LP	$s_\infty = \pm j\Omega$	$m_i = \frac{\sqrt{\Omega^2-1}}{\Omega}$	$0 < m_i \leq 1$	$q_i(s) = \frac{\quad}{\sqrt{s^2+1}}$
	$s_\infty = \pm \Sigma$	$m_i = \frac{\sqrt{\Sigma^2+1}}{\Sigma}$	$1 \leq m_i < \infty$	
	$s_\infty = \Sigma + j\Omega$	$m_i = \frac{\sqrt{(\Sigma+j\Omega)^2+1}}{(\Sigma+j\Omega)} = m_i^{(r)} + j\, m_i^{(i)}$	—	$q_i(s) = \frac{(\lvert m_i\rvert^2+1)s^2+1}{2m_i^{(r)} s\sqrt{s^2+1}}$
	Elementary q'-function for a simple pole at ∞:			$q'(s) = \sqrt{\frac{s+j}{s-j}}$
BP	$s_\infty = \pm j\eta$, $0 \leq \eta < \frac{1}{\sqrt{\tau}}$	$m_i = \sqrt{\frac{\tau - \eta_\infty^2}{\tau^{-1} - \eta_\infty^2}}$	$\tau \leq m_i < \infty$	$q_i(s) = m_i \sqrt{\frac{s^2+\tau^{-1}}{s^2+\tau}}$
	$s_\infty = \pm j\eta$, $\sqrt{\tau} < \eta < \infty$		$0 < m_i < 1$	
	$s_\infty = \pm\sigma$	$m_i = \sqrt{\frac{\sigma^2+\tau}{\sigma^2+\tau^{-1}}}$	$1 < m_i \leq \tau$	
	$s_\infty = \sigma + j\eta$	$m_i = \sqrt{\frac{(\sigma+j\eta)^2+\tau}{(\sigma+j\eta)^2+\tau^{-1}}} = m_i^{(r)} + j\, m_i^{(i)}$	—	$q_i(s) = \frac{\lvert m_i\rvert^2(s^2+\tau\ )+(s^2+\tau)}{(2m_i^{(r)})\sqrt{(s^2+\tau^{\ \ 1})(s^2+\tau)}}$
	Elementary q'-function providing a simple pole at 0 and ∞:			$q'(s) = \sqrt{\frac{\sqrt{\tau}(s^2+1)+j(\tau-1\ )s}{\sqrt{\tau}(s^2+1)-j(\tau^{-1})s}}$

Figure V.10 : q and q' Functions for Lowpass and Bandpass

(c) Composition with the elementary q' functions

$$\frac{q'+1}{q'-1} = \frac{\dfrac{m_1 s}{\sqrt{s^2+1}} + 1}{\dfrac{m_1 s}{\sqrt{s^2+1}} - 1} \quad \frac{\sqrt{\dfrac{s+j}{s-j}} + 1}{\sqrt{\dfrac{s+j}{s-j}} - 1} = \frac{[m_1 s + \sqrt{s^2+1}][\sqrt{s+j} + \sqrt{s-j}]}{[m_1 s - \sqrt{s^2+1}][\sqrt{s+j} - \sqrt{s-j}]}$$

$$= \frac{[m_1 s\sqrt{s+j} + \sqrt{s^2+1}\sqrt{s-j}] + [m_1 s\sqrt{s-j} + \sqrt{s^2+1}\sqrt{s+j}]}{[m_1 s\sqrt{s+j} + \sqrt{s^2+1}\sqrt{s-j}] - [m_1 s\sqrt{s-j} + \sqrt{s^2+1}\sqrt{s+j}]}$$

The composite q'-function is therefore

$$q' = \frac{m_1 s + (s+j)}{m_1 s + (s+j)}\sqrt{\frac{s+j}{s-j}} = \frac{(1.866s - j)}{(1.866s + j)}\sqrt{\frac{s+j}{s-j}}$$

(d) Characteristic function

$$K_o(s) = \mp j\,\frac{(1.865s - j)^2(s+j) + (1.865s + j)^2(s-j)}{(1.865s - j)^2(s+j) - (1.865s + j)^2(s-j)}$$

$$K_o(s) = \pm\,\frac{3.47\,s^3 + 3.73\,s}{0.25\,(s^2 + 4)}$$

Final remarks

In the preceding discussions, the emphasis was on q- and q'-functions for lowpass filters. By a proper selection of the m_i's, the attenuation poles can be selected as pairs on either axis or as conjugate quadruplets or at infinity. The pertinent formulas are compiled in the table of Fig. V.10. This table contains also the various elementary q- and q'-functions for bandpass filters. Filters designed with these functions will display an equal-ripple passband ranging from f_{-p} to f_p. The same equations (V.46), (V.48), (V.56) and (V.60) apply. The pertinent functions are normalized with respect to the center frequency.

$$f_c = \sqrt{f_{-p}\,f_p} = f_{ref} \qquad \text{(V.62)}$$

and employ an auxiliary parameter

$$\tau = \frac{f_p}{f_{-p}} > 1 \qquad \text{(V.63)}$$

The elementary q'-function of the bandpass can be derived from the q'-function of the lowpass by conventional bandpass transformation. Therefore, it will provide a simple pole each at 0 and infinity. The structure of the elementary q-function should be obvious by inspection.

3. Frequency transformation

In some applications it is desirable to modify the characteristic function resulting from methodical approximation, by frequency transformations, or to extend its range of application. These frequency transformations have a threefold purpose:

(a) to obtain a more favourable pole-zero pattern, without changing essentially the performance type.

(b) to change essentially the performance type by transposing the passband and the stopband to different regions.

(c) to generate a higher degree characteristic function from a lower-degree model.

Mathematically, frequency transformations may be considered as conformal mappings of the s-plane to the s_t-plane of a transformed variable by means of

$$s = f(s_t) \tag{V.64}$$

Because the transformation is applied to the e independent variable, any dependent quantity, for instance the transducer loss or the phase, will be transposed according to the mapping properties. However, if the dependent quantity involves a differentiating or integrating process it will not remain identical.

The category of frequency transformations include also the conventional reactance transformations bywhich a conventional lowpass is converted to a highpass, frequency-symmetrical bandpass or bandstop, or even networks with several passbands. For these, $f(s_t)$ must necessarily be a reactance function. The discussion of these transformations will be omitted.

(a) Bilinear Transformations

Some of the standard lowpass approximations do not provide a reflection zero at the origin, or an attenuation pole at infinity. Consequently, one may expect different terminations or coils with mutual coupling or both. Such was the case with the first numerical example in subsection III.2. By means of a bilinear transformation one may transpose one of the reflection zeroes to the origin, one of the attenuation poles to infinity or both. It can be applied whenever the pertinent characteristic functions are even.

The bilinear mapping function and

its inverse are

$$s_t^2 = \frac{a\,s^2 + b}{c\,s^2 + d} \;\underset{\leftarrow}{\rightarrow}\; s^2 = \frac{-d\,s_t^2 + b}{c\,s_t^2 - a} \tag{V.65}$$

In the direction s-plane to s_t-plane, it is convenient to separate the overall transformation into three consecutive steps

$$s_1 = s^2 \;\rightarrow\; s_2 = \frac{a\,s_1 + b}{c\,s_1 + d} \;\rightarrow\; s_t = \sqrt{s_2}$$

The first step maps the ± j-axis of the s-plane onto the negative real axis of the s_1-plane. Significant is the second step which is a bilinear transformation of the real axis onto itself (a,b,c,d are real). This step introduces three parameters and, therefore, permits the selection of three pairs of corresponding arguments. The final step transposes the negative real axis to the ± j-axis of the s_t-plane.

As an example, this transformation will be applied to an 8th degree Cauer lowpass (see Fig. V.11). The top part shows its poles and zeroes in the s_1 (which is the s^2-plane). The transformed patterns of poles and zeroes are shown below.

Part "a": Transformation to a Cauer parameter, case b, lowpass.

Purpose: to transpose one attenuation pole to infinity in order to arrive, potentially, at a filter without negative elements.

corresponding arguments: $s^2 = 0 \;\underset{\leftarrow}{\rightarrow}\; s_t^2 = 0$

$s^2 = -a_1^{-2} \;\underset{\leftarrow}{\rightarrow}\; s_t^2 = \infty$

Part "b": Transformation to a Cauer parameter, case c, lowpass.

Purpose: to transpose one attenuation pole to infinity and one reflection zero to the origin in order to arrive, potentially, at a filter without negative elements and equal terminations.

corresponding arguments: $s^2 = -a_1^2 \;\underset{\leftarrow}{\rightarrow}\; s_t^2 = 0$

$s^2 = -a_1^{-2} \;\underset{\leftarrow}{\rightarrow}\; s_t^2 = \infty$

Part "c": Transformation to a bandpass for single-sideband applications.

Purpose: to transpose one pole each to infinity and to zero to

arrive, potentially, at a circuit without negative elements, and to place all remaining attenuation poles in the upper stopband.

corresponding arguments:

$$s^2 = -a_1^{-2} \rightleftarrows s_t^2 = 0$$

$$s^2 = -a_3^{-2} \rightleftarrows s_t^2 = \infty$$

In all three transformations, the third available parameter becomes an arbitrary and insignificant scaling constant in the s_t-plane. In part a and b, for instanc it is customary to select it such that

$$s^2 = -a_8^2 \rightleftarrows s_t^2 = -1$$

In part c, one may select the parameter k for instance such that the passband limits of the bandpass are reciprocal with respect to each other.

$$s^2 = -a_8^2 \rightleftarrows s_t^2 = -\tau \qquad\qquad s^2 = 0 \rightleftarrows s_t^2 = -\frac{1}{\tau} \qquad\qquad k = -\frac{1}{\tau} = -\sqrt{\frac{1 - a_3^2 a_8^2}{1 - a_1^2 a_8^2}} \qquad \text{(V.66)}$$

Example: Transfer the Cauer parameters of the Glowarzki tables [GL-1]

$a_1 = 0.208540 \quad a_3 = 0.558808 \quad a_5 = 0.772306$

$a_7 = 0.864556 \quad a_8 = 0.875240$

$N = 8,\ \Theta = 50°$

by a "single-sideband" transformation according to Fig. V.11, part c.

equation (V.66) $k = \sqrt{\frac{1 - 0.312266 * 0.766045}{1 - 0.043489 * 0.766045}} = -0.887135$

Therefore,

$$s_t = \pm j \sqrt{|k|} \sqrt{\frac{1 + 0.043489\ s^2}{1 + 0.312266\ s^2}}$$

Applied to all significant frequencies in the s-plane yields the following table of transform pairs:

s-plane	s^2-plane	s_t^2-plane	s_t-plane	
j 0.000000	- 0.000000	- 0.887135	j 0.941878	passband
j 0.298540	- 0.043489	- 0.897648	j 0.947443	
j 0.558808	- 0.312266	- 0.969637	j 0.984702	
j 0.772306	- 0.596456	- 1.061907	j 1.030488	
j 0.864556	- 0.747457	- 1.119624	j 1.058123	
j 0.875240	- 0.766045	- 1.127224	j 1.061707	
j 1.142544	- 1.305406	- 1.412593	j 1.188525	stopband
j 1.156663	- 1.337869	- 1.435036	j 1.197929	
j 1.294823	- 1.676565	- 1.726153	j 1.313831	
j 1.789523	- 3.202293	- ∞	j ∞	
j 4.795243	-22.994354	- 0.000000	j 0.000000	

The same process yields also the parameter values of Fig. V.12. The nomograph on the bottom of this figure may be used to select suitable sets of Cauer parameters if a single-sideband performance is desired. From the design parameters

$$\tau = f_p/f_{-p} \quad \text{and} \qquad \eta_s = \frac{f_s}{\sqrt{f_{-p}f_p}}$$

f_{-p}, f_p = passband limits, f_s = edge of the stopband.

one obtains first the degree and the modular angle. These parameters in combination with the nomograph of Fig. V.6 hield a set of possible pairs of A_{max}(dB) and A_{min}(dB) of the passband and stopband, respectively.

Example: Design the characteristic function of a single-sideband bandpass with a passband range f_{-p} = 95.0 kHz to f_p = 106.0 kHz and a stopband extending from f_s = 119.0 kHz to ∞.

A_{min} = 60 dB.

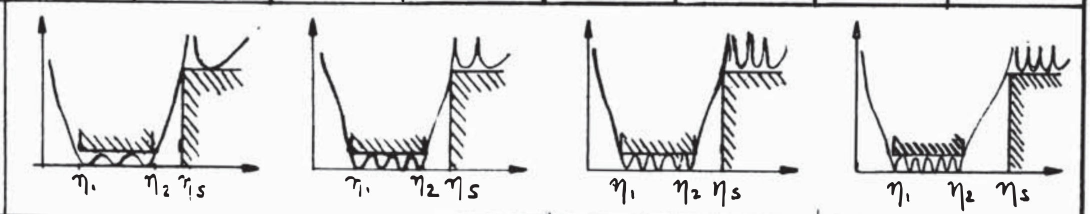
$\tau = f_p / f_{-p} = 1.113$, $f_{ref} = f_{center} = \sqrt{f_{-p}\, f_p} \sim 100$ kHz

$\eta_s = f_s / f_{ref} = 1.19$

Θ	N = 6		N = 8		N = 10		N = 12	
	τ	η_s	τ	η_s	τ	η_s	τ	η_s
5	1.002	1.3660	1.001	1.1796	1.001	1.1085	1.000	1.0731
10	1.007	1.3660	1.004	1.1796	1.003	1.1085	1.002	1.0719
15	1.015	1.3661	1.009	1.1796	1.006	1.1085	1.004	1.0732
20	1.027	1.3663	1.017	1.1798	1.011	1.1086	1.008	1.0732
25	1.044	1.3667	1.027	1.1800	1.018	1.1088	1.013	1.0734
30	1.064	1.3675	1.040	1.1805	1.026	1.1092	1.019	1.0736
35	1.090	1.3688	1.055	1.1814	1.037	1.1098	1.026	1.0741
40	1.122	1.3709	1.075	1.1828	1.050	1.1108	1.035	1.0748
45	1.162	1.3743	1.098	1.1851	1.066	1.1140	1.046	1.0759
50	1.211	1.3794	1.127	1.1885	1.084	1.1147	1.059	1.0776
55	1.272	1.3872	1.163	1.1937	1.107	1.1183	1.075	1.0802
60	1.349	1.3989	1.207	1.2014	1.135	1.1237	1.095	1.0841
65	1.450	1.4165	1.264	1.2131	1.172	1.1318	1.120	1.0900
70	1.587	1.4438	1.340	1.2312	1.219	1.1443	1.153	1.0991
75	1.786	1.4881	1.448	1.2603	1.287	1.1644	1.199	1.1137
80	2.111	1.5670	1.620	1.3117	1.393	1.1999	1.272	1.1395
85	2.810	1.7338	1.972	1.4241	1.608	1.2772	1.417	1.1956

$\eta_1 = \frac{1}{\sqrt{\tau}}$

$\eta_2 = \sqrt{\tau}$



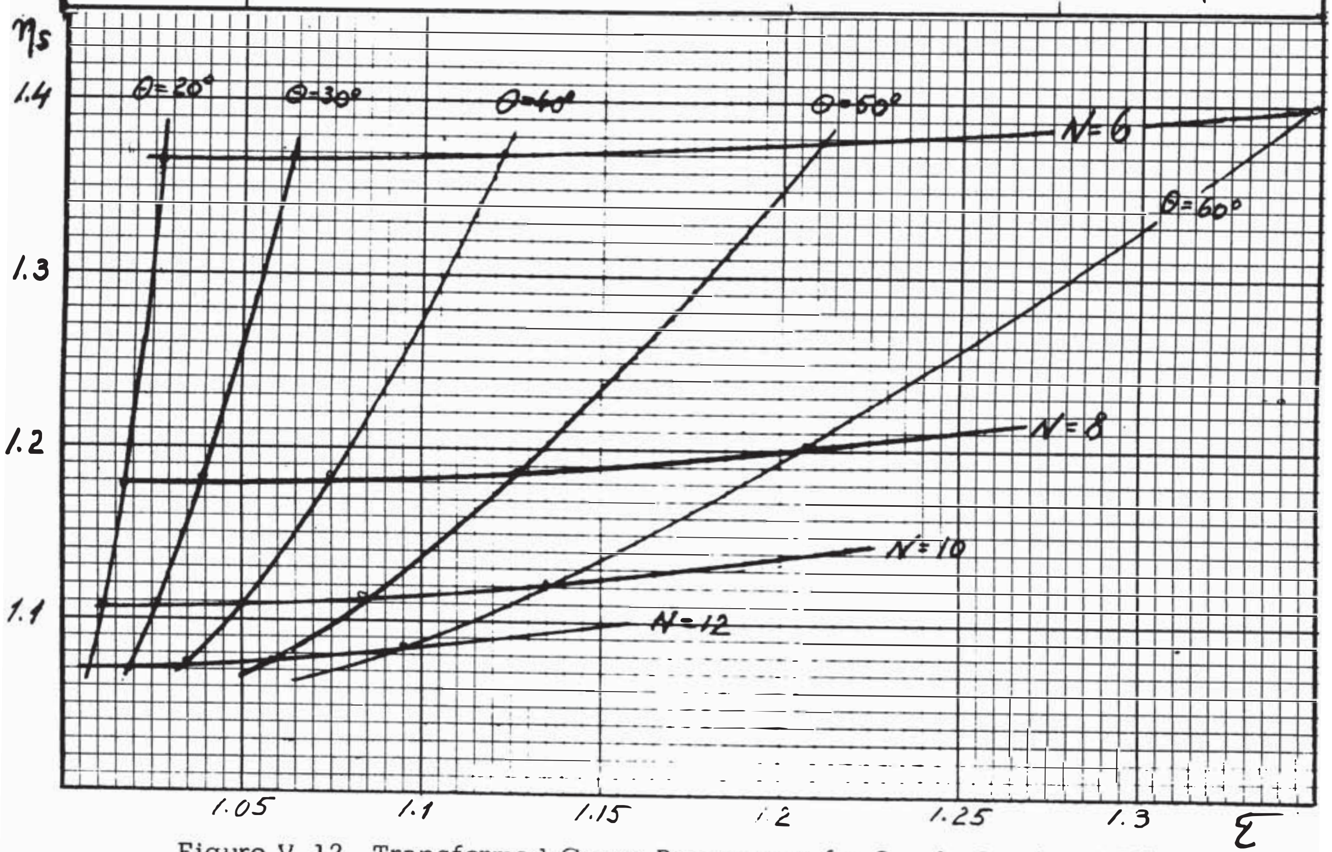


Figure V.12 Transformed Cauer Parameters for Single-Sideband Filters

for this bandpass one may use the parameters derived from the Cauer parameters above. With these the significant frequencies will be

$$f_{-p} = 94.181 \text{ kHz}, \; f_p = 106.171 \text{ kHz}, \; f_s = 118.852 \text{ kHz}.$$

From the intersection of the curve N = 8 and $\Theta = 50°$ in the left nomograph of Fig. V.6 one finds L = 0.006. Moving from the point of intersection to the right and intersecting with "A_{min} = 60 dB" yields an associated "A_{max}=0.005 dB".

Data cards for complete synthesis (SYNTH)

```
COLUMN   1    1    2    2    3    3    4    4    5    5    6    6    7    7    8
.........0....5....0....5....0....5....0....5....0....5....0....5....0....5....0

 COMPLETE SYNTHESIS
 SINGLE SIDEBAND BANDPASS 95-105 KHZ                                      K(S)-DESIGN
 DEGREE=  8  REFL.ZEROES       0 AT THE ORIGIN      4 PAIRS                0 REAL
             ATTEN.POLES       2 AT THE ORIGIN      2 PAIRS                0 QUADS
 REF.FRQ. 100.0   KHZ.    A0=  60.0     DB  AT      118.852 KHZ
 EVAL.    LOSS RESPONSE         LOSS RESPONSE
 FROM      94.0     KHZ          118.0    KHZ
 TO        107.0    KHZ          170.0    KHZ
 SCALE    LINEAR                LINEAR
 WITH      0.25     KHZ INCR.    1.0      KHZ INCR.
PLOT MARGINS
 LEFT      0.0      DB           0.0      DB
 RIGHT     0.1      DB           60.0     DB
 SUBDIV.   0.01     DB           20.0     DB
 REFL.ZEROES RE(KHZ)                IM(KHZ)
 A(1)=     0.0          B(1)=  94.744
 A(2)=     0.0          98.470
 A(3)=     0.0          103.049
 A(4)=     0.0          105.812
 ATTEN.PLS.  RE(KHZ)                IM(KHZ)
 X(1)=     0.0          Y(1)    119.793
 X(2)=     0.0          131.383
 REALIZATION DATA
 REF.RES. 1.0
   2 REALIZATIONS
   1   3   4   5   2   6
   1   4   3   5   2   6

COLUMN   1    1    2    2    3    3    4    4    5    5    6    6    7    7    8
.........0....5....0....5....0....5....0....5....0....5....0....5....0....5....0
```

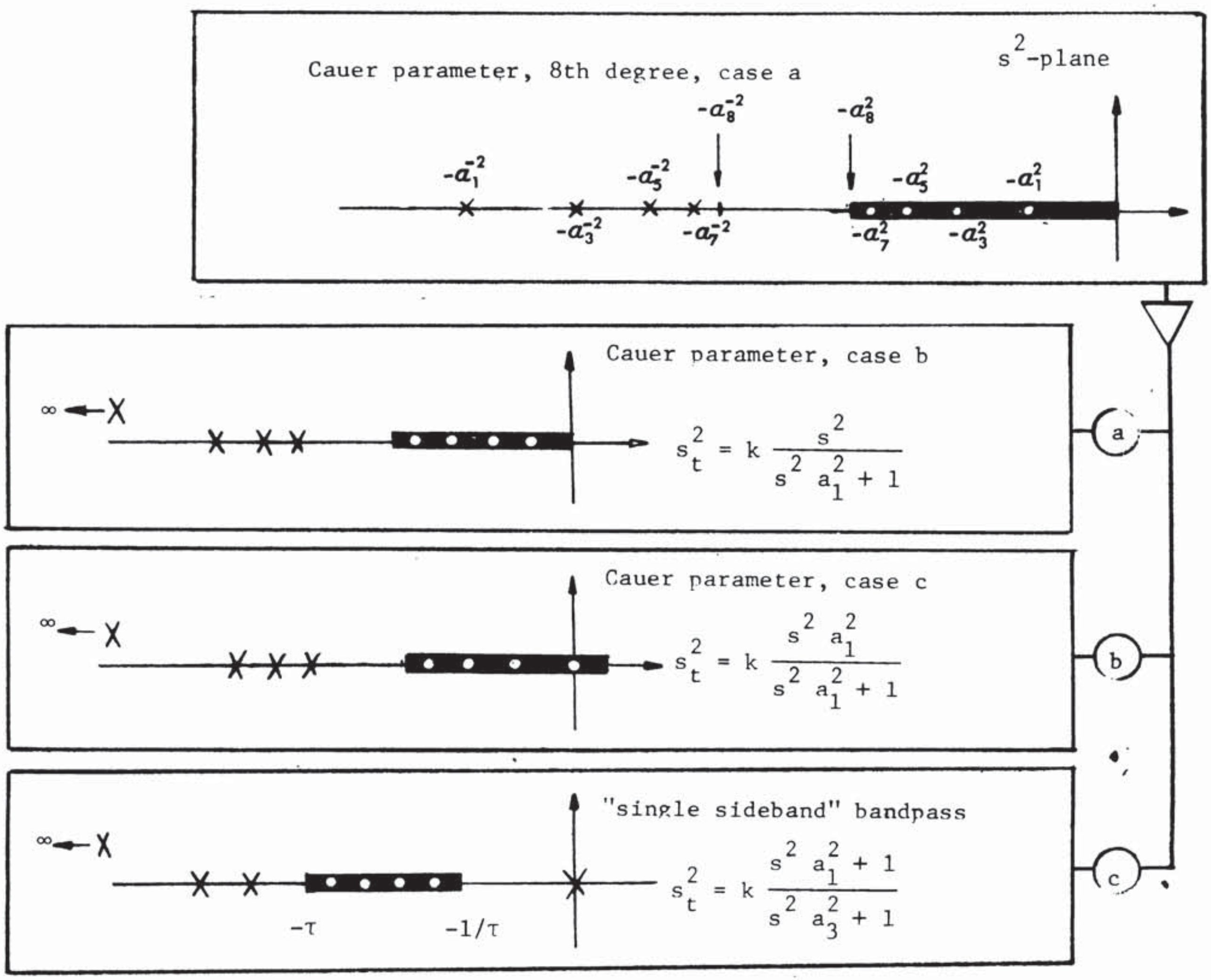
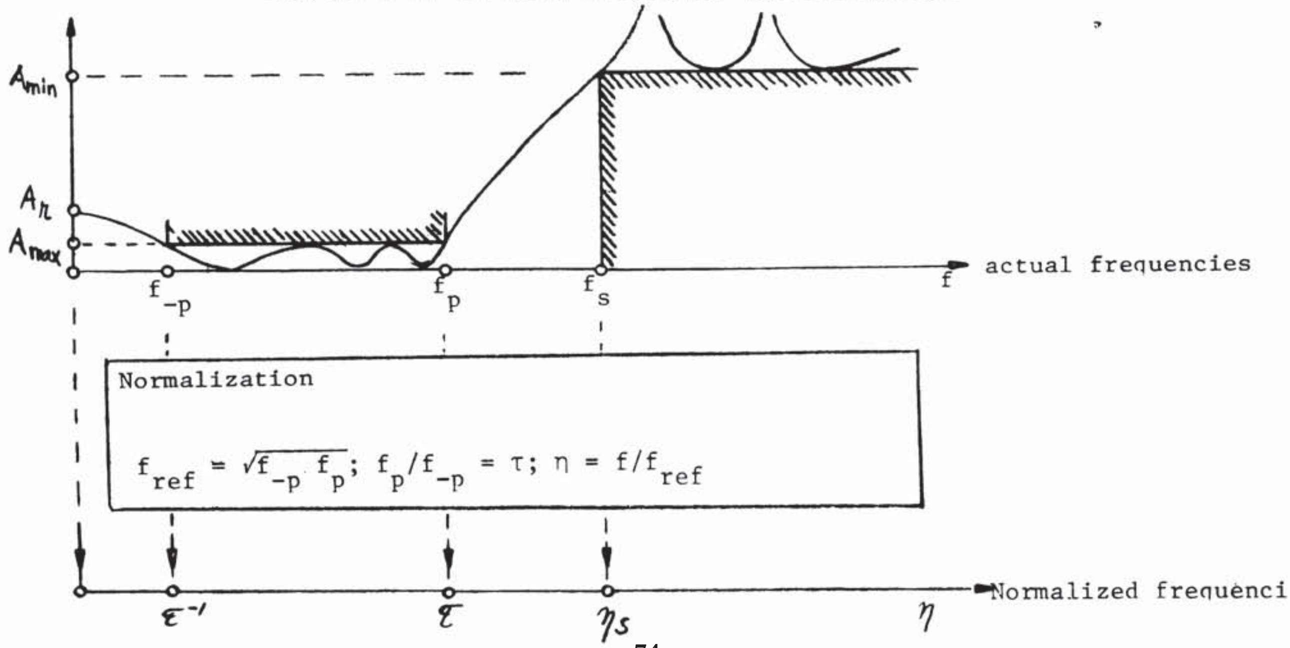


Figure V.11 Bilinear Frequency Transformation



If the stopband happens to be below the passband, take first its reciprocal location with respect to f_{center} and, eventually, use the reciprocal of the resulting pole and zero locations.

By suitable choice of parameters, one may use the bilinear transformation to derive a lowpass performance similar to the one shown in Fig. V.13. Significant for this performance type is a finite loss A_r (dB) at $s = 0$. This transducer loss is uniquely related to the ratio of the terminating resistors. Significant for the performance is also a passband range f_{-p} to f_p with an upper boundary of the transducer loss or the return loss.

A network with a performance as shown in Fig. V.13 may therefore be useful if the objective is a transmission network with a specified return loss in a limited part of the frequency range $0 < f_{-p} < f_p < \infty$ and specified terminations. If there are no selectivity requirements the most economical solution in most cases will be a simple transformer. However, if there are selectivity requirements then a network with the above described performance may be the most economical solution. For further design details see [CE-2] .

(b) Zdunek transformation

Significantly different from the bilinear transformation according to equation (V.65) is a transformation of the following type:

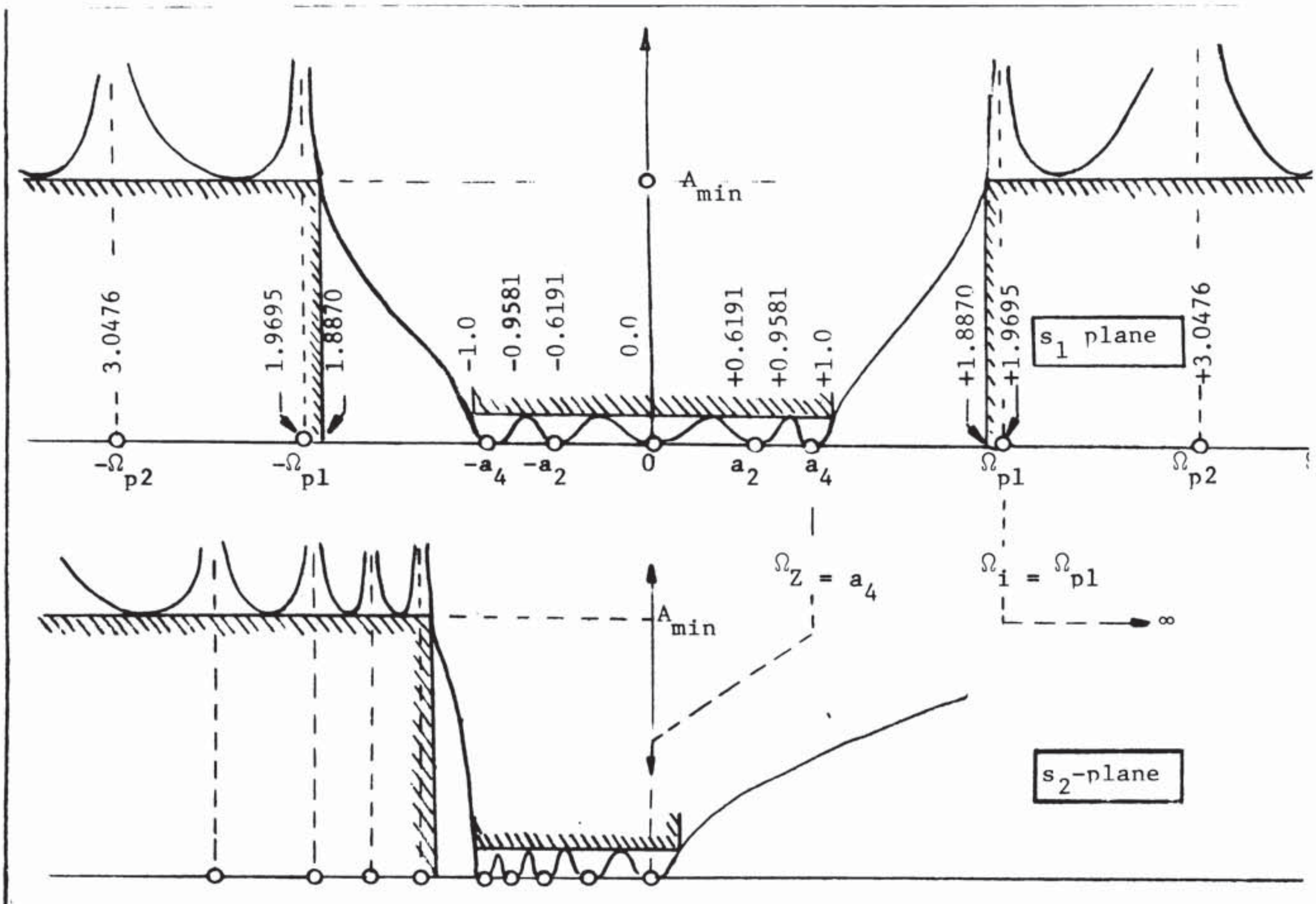
$$s_t^2 = \frac{j\, a\, s + b}{j\, c\, s + d} = k \frac{j\, s + \Omega_z}{j\, s + \Omega_i} \qquad \text{(V.67)}$$

$$a,b,c,d, = \text{real}, \; a,c, \neq 0$$

(Zdunek transformation [ZD-1]). As in the previous transformation, it is again instructive to separate the overall transformation into three consecutive steps

$$s_1 = js \; \rightarrow \; s_2 = \frac{a\, s_1 + b}{c\, s_1 + d} \; \rightarrow \; s_t = \sqrt{s_2}$$

The essential difference to the previous transformation is the first step representing a rotation of the axis by 90°. Therefore, if a characteristic function behaves in a certain way along the imaginary axis of the s_1-plane. The subsequent transformation steps are identical to those of the previous transformation. As previously, the three parameters introduced in the second step govern the actual transposition of poles and zeroes and, therefore, of the performance.



CMEGA(Z) = 0.95813996 CMEGA(I) = 1.96952057 K = 1.23146248

	S-PLANE REAL	S-PLANE IMAG	ST-PLANE REAL	ST-PLANE IMAG
F(S)	0.0	-0.95813996	0.0	0.996[illegible]4[illegible]
	0.0	-0.61919039	0.0	0.96125978
	0.0	0.0	0.0	0.85892522
	0.0	0.61919[illegible]39	0.0	0.61697614
	0.0	0.95813996	0.0	0.0
P(S)	0.0	3.04764843	0.0	1.71438217
		∞	0.0	1.23146248
	0.0	-3.04764843	0.0	1.1[illegible]36182
	0.0	-1.96952[illegible]57	0.0	1.[illegible]165172
	0.0	-1.887[illegible]79[illegible]4	0.0	[illegible]15773449
E(S)	-0.13550103	-1.07306004	-0.[illegible]5397[illegible]12	-0.02467620
	-0.42873269	-0.72958847	-0.80817056	-0.05446788
	-0.60042393	0.0	-0.[illegible]441	-0.11302918
	-0.42873269	0.72958847	-0.[illegible]73[illegible]	-0.22796059
	-0.13550103	1.073[illegible]4	-[illegible]73[illegible]	-0.24527637

Figure V. 14 The Generation of the 10th degree Cauer Filter from a 5th

The Zdunek transformation is especially useful to generate the poles and zeroes of filters of higher order from lower order models. An example is shown in Fig. V.14. In this example the poles and zeroes of a 10th degree Cauer lowpass, type c, are derived from a 5th degree model. The parameters of this model are taken from [CE-1]. The transformation was carried out by a computer program the listing of which is contained in the Appendix D. Additional information regarding further applications of the Zdunek transformation may be found in [CE-1].

4. Approximations based on transducer functions

Whenever the specifications of a transmission network include phase, delay or transient response the design of a characteristic function, initially, is of little or no value. The approximation must start with the poles and zeroes of the transducer function. In most cases, suitable locations for these are found by an optimization procedure by which one or, simultaneously, several specified responses can be approximated. However, in analogy to standard lowpass designs there exist also numerous parameter tables for the parameters of H(s) for certain often used standard response types. Some of these will be discussed briefly.

(a) Maximally flat group delay

The response of phase or delay depends only on the Hurwitz polynomial E(s). For maximally flat group delay, this E(s) must be a Bessel polynomial. An excellent derivation may be found in [BA-2]. According to this reference (with the notation of this chapter)

$$N = 2 \qquad E(s) = 1 + 3\,s + 3\,s^2$$

$$N = 3 \qquad E(s) = 1 + 2 \times 3\,s + 3 \times 5\,s^2 + 3 \times 5\,s^3$$

$$N = 4 \qquad E(s) = 1 + 2 \times 5\,s + 3 \times 3 \times 5\,s^2 + 3 \times 5 \times 7\,s^3 + 3 \times 5 \times 7\,s^4$$

etc.

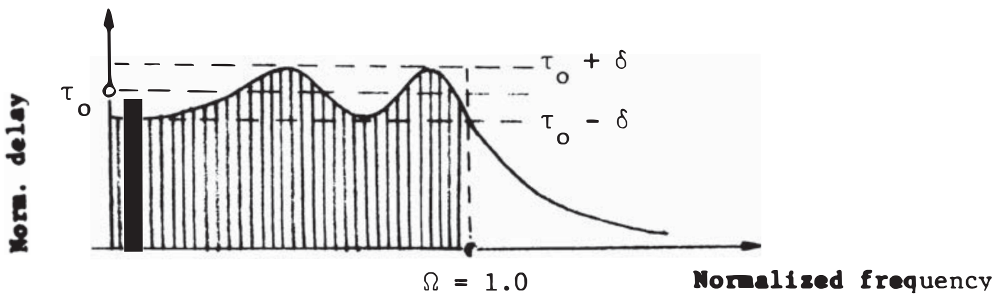
in general

$$E(s) = \sum_{K=0}^{N} b_k s^k = \sum_{K=0}^{N} \frac{(n+k)!}{(n-k)!\,k!} \left(\frac{s}{2}\right)^k \qquad (V.68)$$

(b) Group delay with a Chebyshev variation in the passband range

Parameter tables for this performance were first published by (UP-1) some of these tables may be found in Appendix E.

Performance



The tables in the appendix contain the real and imaginary parts of the natural modes. The quantity "η" is the ratio of the area under the delay curve in the passband range ($0 \leq \Omega \leq 1$) to the total area. This quantity is constant for any given degree.

Example: Design a lowpass with 55 μsec ±1.5% group delay in the range 0 - 10 kHz

$$f_{ref} = 10 \text{ kHz} \qquad T_{ref} = (2\pi f_{ref})^{-1} = 15.9\ \mu_{sec}$$

$$\tau_o = \frac{55.0}{15.9} = 3.45; \qquad \delta = 0.015\ \tau_o = 0.051$$

In Appendix E, the tables contain the parameters for allpass sections which provide twice the delay. [Reason: $E^2(s)$ rather than $E(s)$ in the numerator of $E(s)$.] Therefore, one must look up

$$2\ \tau_o = 6.9 \quad \text{and} \quad 2\ \delta = 0.102$$

Suitable parameters in the table: $N = 4$; $\tau_o = 6.847$; $\delta = 0.1$

$a_{1,2} = 0.548547$; $b_{1,2} = \pm\ j\ 0.341938$

$a_{3,4} = 0.442596$; $b_{3,4} = \pm\ j\ 0.593948$

In both filters, (a) and (b), attenuation poles can be added to improve the stopband performance.

(c) Design for transient response

In some practical applications, an important design objective for a transmission network is the transient response at the output resulting from an input signal with specified wave form. The methods to solve problems of this type are the subject of time-domain synthesis which is beyond the scope of this chapter.

However, as an aid to designers several authors have carried out the pertinent calculations for the most commonly used wave forms. The resulting poles and zeroes are available in several publications, for instance [JS-1] and also in book form [JE-1] . With these as input data, one may carry out a large number of designs for good transient performance.

The most commonly used input wave forms are the unit step and the unit impulse for which the transient response can be calculated by the inverse Laplace transformations

$$\left.\begin{aligned} h(t) &= \mathcal{L}^{-1}\left[\frac{1}{H(s)}\right] = \mathcal{L}^{-1}\left[\frac{P(s)}{E(s)}\right] && \text{for impulse response} \\ g(t) &= \mathcal{L}^{-1}\left[\frac{1}{sH(s)}\right] = \mathcal{L}^{-1}\left[\frac{P(s)}{sE(s)}\right] && \text{for step response} \end{aligned}\right] \quad (V.69)$$

For the H(s) of a lowpass, typical time responses are shown in Fig. V.15. Especially for data transmission, it is most desirable to obtain.

(a) A rather rapid rise or fall of the output voltage whenever a signal appears or disappears at the input; for this, the bandwidth of the lowpass should be as large as possible.

(b) insensitivity with respect to noise and undesired signals; for this, the bandwidth should be as narrow as possible.

(c) at the trailing ends, the over and undershoots must stay below specified limits.

Obviously from these objectives, the poles and zeroes of H(s) must be selected such that the resulting network satisfies tolerance requirements in the frequency and in the time domain. Also, as in many design problems it is economical to make full use of the permitted tolerances.

The table of Fig. V.16 contains the poles and zeroes of H(s) of lowpass filters with optimized transient response. It is a reprint of [JS-1] . By the number in the left-most column, the performance is identified in the following manner:

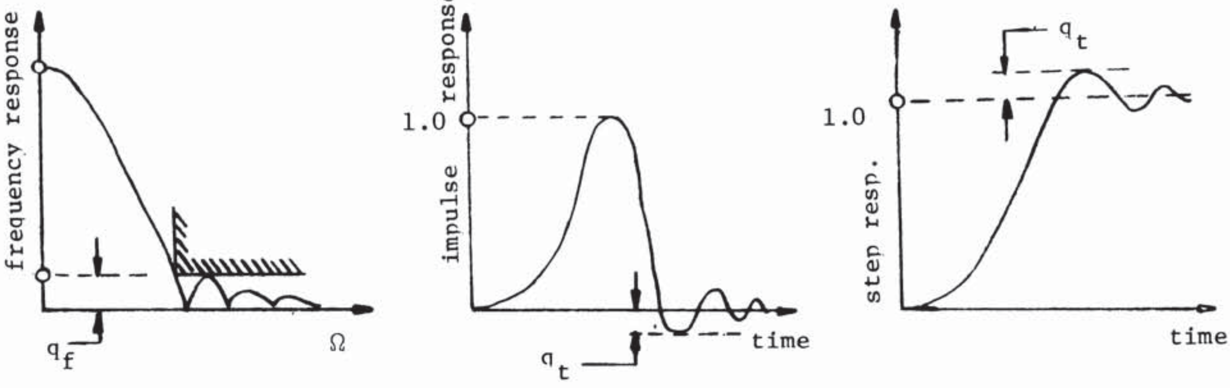


Figure V. 15   Typical Frequency and Time Response of a Low-Pass

$$A(p') = K\,\frac{\prod_{\mu=1}^{m/2}(p'-p'_{0\mu})(p'-p'^{*}_{0\mu})}{\prod_{\nu=1}^{n/2}(p'-p'_{\infty\nu})(p'-p'^{*}_{\infty\nu})},\qquad A(p') = K\,\frac{\prod_{\mu=1}^{m/2}(p'-p'_{0\mu})(p'-p'^{*}_{0\mu})}{(p'-p'_{\infty 1})\prod_{\nu=2}^{(n+1)/2}(p'-p'_{\infty\nu})(p'-p'^{*}_{\infty\nu})}$$

(m gerade, n gerade), (m gerade, n ungerade).

	K	σ'_{01}	ω'_{01}	σ'_{02}	ω'_{02}	$\sigma'_{\infty 1}$	$\omega'_{\infty 1}$	$\sigma'_{\infty 2}$	$\omega'_{\infty 2}$	$\sigma'_{\infty 3}$	$\omega'_{\infty 3}$	M
42.05 : D	0,0214957	0	1,102130	–	–	– 0,275533	0,148650	– 0,236820	0,458597	–	–	2,683
42.10 : D	0,0448750	0	1,086302	–	–	– 0,301837	0,194652	– 0,241771	0,593351	–	–	2,053
42.20 : D	0,0756796	0	1,086107	–	–	– 0,327690	0,230920	– 0,229062	0,709258	–	–	1,678
52.05 : D	0,0269420	0	1,065391	–	–	– 0,327219	0	– 0,300249	0,319768	– 0,227973	0,658610	2,230
52.10 : D	0,0485792	0	1,060048	–	–	– 0,354624	0	– 0,323474	0,372352	– 0,217233	0,765212	1,934
52.20 : D	0,0852998	0	1,059987	–	–	– 0,445148	0	– 0,372144	0,386821	– 0,196644	0,841776	1,652
62.05 : D	0,0229360	0	1,047547	–	–	– 0,302917	0,162267	– 0,270929	0,484546	– 0,180611	0,811760	2,202
62.10 : D	0,0357719	0	1,042081	–	–	– 0,320779	0,174728	– 0,271067	0,530792	– 0,142811	0,893561	1,918
42.05 : S	0,0202716	0	1,104121	–	–	– 0,272811	0,149424	– 0,196288	0,465716	–	–	2,040
42.10 : S	0,0423915	0	1,088113	–	–	– 0,314238	0,184574	– 0,189329	0,584861	–	–	1,542
42.20 : S	0,0757781	0	1,078048	–	–	– 0,343306	0,227722	– 0,153200	0,703880	–	–	1,165
52.05 : S	0,0227196	0	1,069278	–	–	– 0,310756	0	– 0,274242	0,325361	– 0,164700	0,659190	1,744
52.10 : S	0,0413674	0	1,057034	–	–	– 0,342581	0	– 0,291194	0,376463	– 0,123843	0,761764	1,394
52.20 : S	0,0701896	0	1,047305	–	–	– 0,383966	0	– 0,316772	0,429197	– 0,758333	0,835990	1,106
62.05 : S	0,0230172	0	1,047418	–	–	– 0,351490	0,147028	– 0,272405	0,454863	– 0,117398	0,777847	1,628
62.10 : S	0,0383183	0	1,038972	–	–	– 0,377877	0,162532	– 0,279629	0,508736	– 0,074631	0,848409	1,353
62.20 : S	0,0705245	0	1,031012	–	–	– 0,437240	0,182911	– 0,319185	0,564564	– 0,038477	0,889915	1,091
64.05 : S	0,0360007	0	1,026939	0	1,340319	– 0,426413	0,181418	– 0,334308	0,546150	– 0,101188	0,874275	1,574
64.10 : S	0,0718094	0	1,019823	0	1,294457	– 0,490663	0,208169	– 0,366035	0,624222	– 0,045632	0,916068	1,303

Figure V. 16   Poles and Zeros fpr a Transient Response

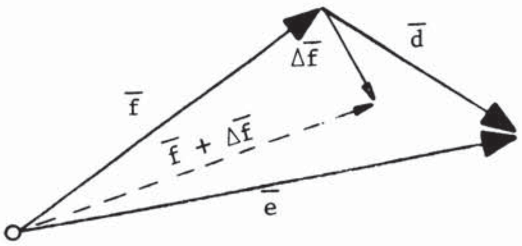


Figure V. 17 The Vectors of the Desi and Actual Response

N M . op : X

$\left\{\begin{array}{l}\text{S = optim. for step response} \\ \text{D = optim. for impulse response ("Dirac")}\end{array}\right.$

boundary for stopband and overshoot as shown in Fig. V.15:
for instance,
op = 10, $q_t = q_f = 0.01$ which means that the overshoot is 1/100 of the maximum impulse response or steady state value, respectively, and the frequency response at the normalized stopband edge $\Omega = 1$ is 1/100 of the amplitude at dc.

degree of P(s)

degree of E(s)

For instance the network with the identification 52.10 : S with the transducer function

$$H(s)=(0.0413674)^{-1}\,\frac{(s + 0.342581)(s^2+ 0.582388s + 0.141724)(s^2+ 0.247686s + 0.595621)}{(s^2+ 1.057034^2)}$$

yields a lowpass with optimized step response, overshoots $\leq$ 1% and a stopband attenuation of 40 dB.

5. Least-square approximations

Standard approximation methods are restricted to a limited number of frequently encountered tolerance plots. For most other tolerance plots, for instance those which require the matching of specified response curves, the approximation is carried out most economically by optimization methods. The example in subsection V.6 is such a case. Of several different optimization methods the least square method will be discussed briefly.

For i = 1,2, ... n, let w_i be a set of n arguments, for instance frequencies, and e_i a set of n desired response values for the transmission network. In most cases, the e_i's result from empirical measurements. Also, let $f(w; p_1, p_2, \ldots p_k)$ be a function representing the response of the contemplated network. For instance $f(w; p_1, p_2, \ldots p_k)$ could be the transducer loss of a 6th degree lowpass if such a lowpass were contemplated as the desired transmission network. In the function $f(w; p_1, \ldots p_k)$, w is the argument, normally the frequency, and p_1, $p_2 \ldots p_k$ a set of k parameters, for instance the reflection zeroes, attenuation poles etc. The initial values for these parameters may be selected by intuition or from suitable tables or by a combination of both.

At the n arguments w_i, $f(w; P_1, P_2 \ldots P_k)$ assumes the n values f_i, i = 1, 2, ... n

$$f_i = f(w_i ; P_1, P_2, \ldots P_k) \tag{V.70}$$

Therefore, the f_i are the actual response values for a set of parameters at the arguments w_i. The two sets of numbers e_i and f_i may be considered to be the n coordinates of two n-dimensional vectors

$$\bar{f} = \begin{bmatrix} f_1 \\ f_2 \\ \cdot \\ \cdot \\ f_n \end{bmatrix} \qquad \bar{e} = \begin{bmatrix} e_1 \\ e_2 \\ \cdot \\ \cdot \\ e_n \end{bmatrix} \tag{V.71}$$

By variation of the parameters, it is the objective of the optimization procedure to make $\bar{f} = \bar{e}$. However, one must expect that the two vectors will differ by a vector $\bar{d}$

$$\bar{d} = \bar{e} - \bar{f} \tag{V.72}$$

as shown in Fig. V.17.

By a small variation of the k parameters, $\bar{f}$ will change to $\bar{f} + \Delta\bar{f}$ where $\Delta\bar{f}$ is approximately

$$\Delta\bar{f} = \sum_{j=1}^{k} \begin{bmatrix} \left.\frac{\partial f}{\partial P_j}\right|_{\omega=\omega_i} \\ \cdot \\ \cdot \\ \cdot \\ \left.\frac{\partial f}{\partial P_j}\right|_{\omega=\omega_n} \end{bmatrix} \Delta P_j = \begin{bmatrix} f_{11}\Delta P_1 + f_{12}\Delta P_2 + \ldots & f_{1k}\Delta P_k \\ f_{21}\Delta P_1 + f_{22}\Delta P_2 + \ldots & f_{2k}\Delta P_k \\ \ldots\ldots\ldots\ldots\ldots\ldots & \\ f_{n1}\Delta P_1 + f_{n2}\Delta P_2 + \ldots & f_{nk}\Delta P_k \end{bmatrix} \tag{V.73}$$

in the rightmost expression, the quantities f_{ij}

$$f_{ij} = \frac{\partial f(\omega_i ; P_1, P_2 \ldots P_k}{\partial P_j} \tag{V.74}$$

are the partial derivatives of $f(\omega;\ P_1, P_2, \ldots P_k)$ for the parameter P_j, evaluated for $\omega = \omega_i$. Ideally, one wants to achieve

$$\Delta\overline{f} = \overline{d} \qquad \Delta\overline{f} - \overline{d} = 0 \tag{V.75}$$

which is equivalent to the following system of linear equations:

$$\begin{aligned}
f_{11}\,\Delta P_1 + f_{12}\Delta P_2 + \ldots \quad & f_{1k}\Delta P_k = d_1 = e_1 - f_1 \\
f_{21}\,\Delta P_1 + f_{22}\Delta P_2 + \ldots \quad & f_{2k}\Delta P_k = d_2 = e_2 - f_2 \\
\ldots\ldots\ldots\ldots \\
f_{n1}\Delta P_1 + f_{n2}\Delta P_2 + \ldots \quad & f_{nk}\Delta P_k = d_n = e_n - f_n
\end{aligned} \tag{V.76}$$

For $n = k$, it possible to solve these equations for the unknown parameter changes ΔP_j and modify all parameters accordingly. Furthermore, it is also conceivable that after several iterations $\overline{f} = \overline{e}$, which means that the response function of the network is equal to the desired response at all arguments w_i. In between the arguments, the actual response may deviate excessively.

To overcome this, it is necessary to make $n > k$ (in practice $n = 5k$, at least) in which case (V.76)contains more equations than unknowns. An exact solution is no longer possible. However, instead of equation (V.75) $(\Delta\overline{f} - \overline{d}) = 0$ one may postulate

$$[\Delta\overline{f} - \overline{d}]^2 = \text{Min!} \tag{V.77}$$

which is equivalent to

$$\begin{aligned}
&[f_{11}\,\Delta P_1 + f_{12}\Delta P_2 \ldots + f_{1k}\Delta P_k - d_1]^2 + \\
&[f_{21}\,\Delta P_1 + f_{22}\Delta P_2 \ldots \quad f_{2k}\Delta P_k - d_2]^2 + \\
&\ldots\ldots\ldots\ldots\ldots \\
&[f_{n1}\Delta P_1 + f_{n2}\Delta P_2 \ldots \quad f_{nk}\Delta P_k - d_n]^2 = \text{Min!}
\end{aligned} \tag{V.78}$$

For a (local) minimum, it is necessary that, simultaneously,

$$\frac{\partial}{\partial(\Delta P_1)} \ldots = 0; \quad \frac{\partial}{\partial(\Delta P_2)} \ldots = 0; \quad \ldots\ldots \quad \frac{\partial}{\partial(\Delta P_k)} \ldots = 0 \tag{V.79}$$

For instance,

$$\frac{\partial}{\partial(\Delta P_j)}\left\{\ \right\} = \begin{array}{ll} 2[(f_{11}\Delta P_1 + f_{12}\Delta P_2 + \cdots & f_{1k}\Delta P_k) - d_1]f_{1j} + \\ 2[(f_{21}\Delta P_1 + f_{22}\Delta P_2 + \cdots & f_{2k}\Delta P_k) - d_2]f_{2j} + \\ \cdots\cdots & \\ 2[(f_{n1}\Delta P_1 + f_{n2}\Delta P_2 + \cdots & f_{nk}\Delta P_k) - d_n]f_{nj} = 0 \end{array} \tag{V.80}$$

$$\begin{bmatrix} f_{11}f_{1j}\Delta P_1 + f_{12}f_{1j}\Delta P_2 + \cdots & f_{1k}f_{1j}\Delta P_k + \\ f_{21}f_{2j}\Delta P_1 + f_{22}f_{2j}\Delta P_2 + \cdots & f_{2k}f_{2j}\Delta P_k + \\ \cdots\cdots & \\ f_{n1}f_{nj}\Delta P_1 + f_{n2}f_{2j}\Delta P_2 + \cdots & f_{nk}f_{nj}\Delta P_k \end{bmatrix} = \begin{bmatrix} d_1 f_{1j} \\ d_2 f_{2j} \\ \\ d_n f_{nj} \end{bmatrix} \tag{V.81}$$

$$C_{1j}\Delta P_1 + C_{2j}\Delta P_2 + \cdots \quad C_{kj}\Delta P_k = D_j \tag{V.82}$$

After carrying out the procedure for all parameters one obtains the following system of k equations:

$$\begin{array}{ll} C_{11}\Delta P_1 + C_{12}\ \Delta P_2 + \cdots\cdots & C_{1k}\ \Delta P_k = D_1 \\ C_{21}\Delta P_1 + C_{22}\ \Delta P_2 + \cdots\cdots & C_{2k}\ \Delta P_k = D_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ C_{k1}\Delta P_1 + C_{k2}\ \Delta P_2 + \cdots\cdots & C_{kk}\ \Delta P_k = D_k \end{array} \tag{V.83}$$

where

$$C_{ij} = C_{ij} = \sum_{v=1}^{n} f_{vi}\ f_{vj}; \qquad D_j = \sum_{v=1}^{n} d_v\ f_{vj} \tag{V.84}$$

The solutions of this linear system of equation are the parameter changes ΔP_j. After the parameters have been modified accordingly, the procedure should be repeated.

If the necessary precautions are taken, the iterative procedure yields in general a set of parameters at a local minimum of equation (V.77). In some applications one may also be reasonably sure that the local minimum is also the global minimum. This is the case when the initial set of parameters were already close the optimum.

Normally, the deviations of the actual from the desired response are not equal-ripple but deviate increasingly as one approaches the edges of the approximation range. To overcome this one may take the following steps:

(a) introduce a weighting factor for the d_i, or

(b) emphasize the edges by a crowing of the argument values towards the edges, or

(c) use the power 2n rather than 2 in equation (V.77).

VI. Transmission Networks for Arbitrary Terminations

All previous derivations and formulas, especially the two-port parameters of Figure III.3 apply only to networks which are terminated by R_{ref} on both sides. However, quite frequently it is desirable to design networks between specified different impedances. In highpass and bandpass filters, this can often be accomplished without the addition of a transformer, by the conversion of a shunt coil. Furthermore, many bandpass circuits lend themselves to impedance changes by one or several suitable network transformations.

Neither of these methods is applicable to lowpass filters. However, different terminations may be obtained by the introduction of "flat loss". The concept is to raise the transducer loss of a suitable reference filter such that its value at s = 0 becomes A_o, where

$$A_o = 10 \log \left[\tfrac{1}{2} \left(\sqrt{R_{gen}/R_{load}} + \sqrt{R_{load}/R_{gen}} \right) \right]^2 \qquad \text{(VI.1)}$$

with R_{gen} = desired generator impedance
R_{load} = desired load impedance

Although this method is of practical value only for lowpass (and highpass) filters it is feasible to apply it also to other networks. It also provides a transition from networks with equal terminations to those where one of these assumes the values 0 or ∞.

1. Design method for flat loss

Let $K_1(s)$ represent the characteristic function of an arbitrary reference lowpass

$$K_1(s) = C_1 \frac{F_1(s)}{P_1(s)} \qquad \text{(VI.2)}$$

From this one may derive

$$\text{(II.21)} \rightarrow \quad H_1(s)H_1(-s) = 1 + K_1(s)\, K_1(s) \qquad \text{(VI.3)}$$

$$\text{(II.14)} \rightarrow \quad E_1(s)E_1(-s) = F_1(s)F_1(-s) + C_1^{-2}\, P_1(s)P_1(-s) \qquad \text{(VI.4)}$$

$$\text{(II.17)} \rightarrow \quad H_1(s) = C_1 \frac{E_1(s)}{P_1(s)} \qquad \text{(VI.5)}$$

$$\text{(II.16)} \qquad A_1[dB] = 10 \log \left[H_1(s)\, H_1(-s) \right]_{s=j\omega} \qquad \text{(VI.6)}$$

Let $F_1(s)$ and $P_1(s)$ be normalized polynomials and let it be also assumed (for tutorial reasons only) that the reflection zeroes are on the j-axis. At these, the normalized driving point impedances are both 1.0. If the normalized load were changed from its original value to an arbitrary value r then by Thevenin's principle the transducer loss at the reflection zeroes would increase by

$$\Delta A[dB] = 10\log_{10}\left[\frac{(1+r)^2}{4r}\right] = 10\log\left[\frac{1}{2}\left(\sqrt{r}+\frac{1}{\sqrt{r}}\right)\right]^2 \qquad \text{(VI.7)}$$

$$\Delta A[dB] = 10\log\gamma^2 \qquad \text{(VI.8)}$$

$$\gamma = \frac{1}{2}\left(\sqrt{r}+\frac{1}{\sqrt{r}}\right) \geq 1 \qquad \text{(VI.9)}$$

At other frequencies, the transducer loss will change by some other frequency dependant amount.

The next step is to consider a transmission network with a transducer loss

$$A[dB] = A_1[dB] + \Delta A[dB] \qquad \text{(VI.10)}$$

where ΔA is a frequency independent flat loss. Its value depends on the number in dB by which A_1 must be incremented to achieve A_c for s = 0. For instance, let the reference lowpass have a transducer loss as shown in Fig. VI.1; furthermore, let R_{gen} = 600 Ohms and R_{load} = 3000 Ohms. Then

$$A_o = 10 \log [\tfrac{1}{2}(\sqrt{3000/600} + \sqrt{600/3000})]^2 = 2.55\text{dB}$$

$$A_1 = 0 \text{ dB} \qquad \Delta A = A_o = 2.55\text{dB}$$

Obviously in this example,

$$r = 3000/600 = 5; \; \gamma = \tfrac{1}{2}[\sqrt{r} + 1/\sqrt{r}] = 1.34164$$

The transducer function H(s) of the desired network is easily derived:

$$\left.\begin{array}{l}(VI.6) \rightarrow A_1 = 10 \log \; H_1(s)H_1(-s)_{s=j\omega} \\ (VI.8) \rightarrow \Delta A = 10 \log \; \gamma^2\end{array}\right\} \quad A = A_1 + \Delta A = 10 \log[\gamma \; H_1(s) \; \gamma H_1(-s)]_{s=j\omega}$$

therefore,

$$H(s) = \gamma H_1(s) = \gamma C_1 \frac{E_1(s)}{P_1(s)} \tag{VI.11}$$

By the introduction of an auxiliary ideal transformer, the desired network can be related to a network with equal terminations, assuming that this network includes the transformer (see Fig. VI.2).

In order to calculate the two-port parameters one must first establish the characteristic function.

(VI.3): $\gamma^2 \; H_1(s) \; H_1(-s) = \gamma^2 + \gamma^2 K_1(s) \; K_1(-s)$

(VI.11): $H(s) \; H(-s) = 1 + [\gamma^2 - 1 + \gamma^2 \; K_1(s) \; K_1(-s)]$

(II.21): $H(s) \; H(-s) = 1 + \qquad K(s) \; K(-s)$

Therefore, $K(s) \; K(-s) = (\gamma^2 - 1 \; + C_1^2 \dfrac{F_1(s) \; F_1(-s)}{P_1(s) \; P_1(-s)}$

$$K(s) \; K(-s) = \gamma^2 \; C_1^2 \frac{F_1(s) \; F_1(-s) + C_2^{-2} \; P_1(s)P_1(-s)}{P_1(s) \; P_1(-s)} \tag{VI.12}$$

$$\text{with} \quad C_2 = C_1 \sqrt{\frac{\gamma^2}{\gamma^2-1}} \tag{VI.13}$$

Consequently,

$$K(s) = \pm\gamma \; C_1 \frac{F(s)}{P(s)} \tag{VI.14}$$

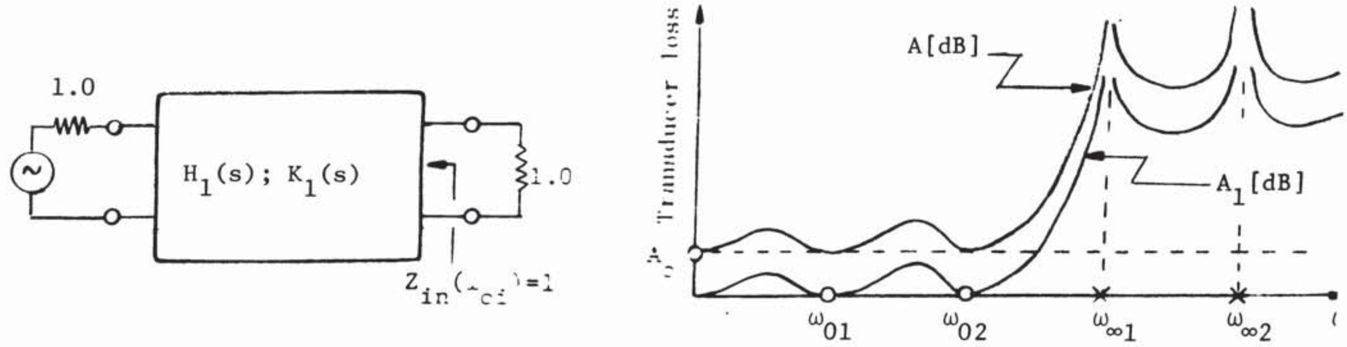


Figure VI.1 The Transducer Loss of the Reference and the Desired Low-Pass

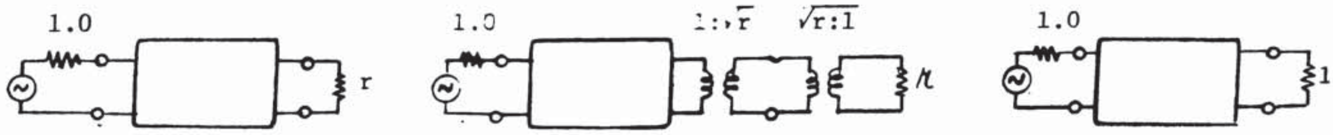


Figure VI.2 Introduction of an Auxiliary Transformer

where the roots of F(s), the new reflection zeroes, are defined by the root pattern of

$$F(s)\ F(-s) = F_1(s)\ F_1(-s) + C_2^{-2}\ P_1(s)\ P_1(-s) \tag{VI.15}$$

This equation is formally identical to equation (VI.4). Consequently, one may consider the new reflection zeroes as natural modes of a secondary reference filter with a characteristic function

$$K_2(s) = C_2\,\frac{F_1(s)}{P_1(s)} \tag{VI.16}$$

However, contrary to the selection of natural modes, the roots of F(s) may be selected either from the right or the left half plane. Different selections yield different transmission networks regarding element values and driving point impedances. All these different networks have the same transfer properties.

Example: Design a lowpass for generator impedance R_{gen} = 600 Ohms and a load impedance R_{load} = 3000 Ohms. The desired lowpass shall have the same passband ripple cut-off rate and stopband discrimination as the following equally-terminated reference lowpass of 5th degree:

normalized reflection zeroes: $0, \pm j\ 1.0, \pm j\ 2.0$

normalized atten. poles $\pm j\ 3.0, \pm j\ 4.0$

A_o = 50.0 dB at $\pm j\ 3.4$,

(a) characteristic function of the reference lowpass

$$K_1(s) = 13.24221\,\frac{s\ (s^2 + 1)(s^2 + 4)}{(s^2 + 9)(s^2 + 16)}$$

From this one determines

$$E_1(s) = s^5 + 2.724999s^4 + 8.709958s^3 + 14.06758s^2 + 16.75988s + 10.8744$$

(b) calculation of the constant C_2

$$R_{load}/R_{gen} = r = 5 \rightarrow \gamma = \tfrac{1}{2}(\sqrt{r} + 1/\sqrt{r}) = 1.34164;\ \gamma^2 = 1.8$$

$$C_2 = C_1\sqrt{\frac{\gamma^2}{\gamma^2-1}} = 19.863116$$

(c) reflection zeroes of the desired network

$$K_2(s) = 19.863116 \frac{s\,(s^2+1)(s^2+4)}{(s^2+9)(s^2+16)}$$

From this one determines the roots of a fictitious Hurwitz polynomial $E_2(s)$; these are

$$u_1 = -0.9266033 + j\,0.0$$
$$u_{2,3} = -0.5435560 \pm j\,1.2589397$$
$$u_{4,5} = 0.1054266 \pm j\,2.0370735$$

these roots or their counterparts in the right half plane are the new reflection zeroes.

One could select all of the left hand roots in which case

$$F(s) = E_2(s) = s^5 + 2.22457s^4 + 7.473085s^3 + 10.72988s^2 + 7.2496$$

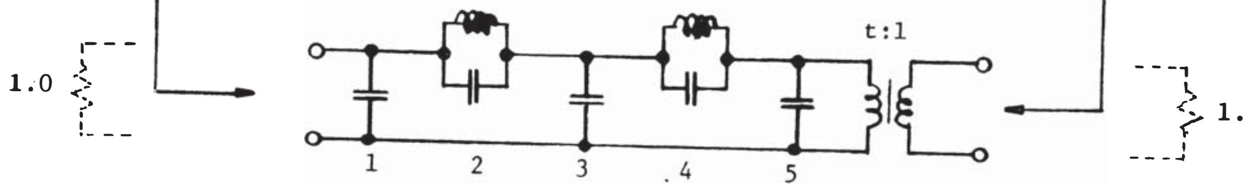
(d) two-port parameters

$$E(s) = E_1(s) \qquad E_e = 2.72499s^4 + 14.06758s^2 + 10.8744$$
$$E_o = s^5 + 8.70995s^3 + 16.7598s$$
$$F(s) = E_2(s) \qquad F_e = 2.22457s^4 + 10.72988s^2 + 7.2496$$
$$F_o = s^5 + 7.47308s^3 + 12.382s$$

The following anticipates the realization of a ladder circuit which will be the subject of the next section.

$$z_{11} = \frac{E_e - F_e}{E_o + F_o} = \frac{0.50042s^4 + 3.3370\,s^2 + 3.6248}{2\,s^5 + 16.1830\,s^3 + 28.980\,s}$$

$$z_{22} = \frac{E_e + F_e}{E_o + F_o} = \frac{4.94956\,s^4 + 24.7975\,s^2 + 18.1240}{2\,s^5 + 16.830\,s^3 + 28.980\,s}$$



The actual element values for this particular selection of reflection zeroes and also for three other selections are compiled in the table of Fig. VI.3. These examples show that the terminating resistor may turn out to be the desired r or its reciprocal. If the reciprocal results the designer can choose between several remedies:

(a) Reverse the direction of transmission: make "r" the normalized generator impedance and "1" the normalized load.

(b) Use the dual structure.

(c) Move an odd number of reflection zeroes to the opposite half plane. Obviously this is only possible if there is one or several roots on the real axis.

A few comments regarding this last point. At s = 0 in a lowpass, the input impedance becomes equal to the load impedance

$$(III.2) \rightarrow r = z_{in}(0) = \left. \frac{E(s) - F(s)}{E(s) + F(s)} \right|_{s=0} = \frac{e_o - f_o}{e_o + f_o} \qquad (VI.17)$$

$$F(s) = \sum_v f_v s^v = \prod_i (s - a_i) \prod_j (s^2 - p_j s + q_j) \qquad (VI.18)$$

Therefore,

$$f_o = \prod_i (-a_i) \prod_j (q_j)$$

The transposition of a root pair to the other plane does not alter the sign of q_j and, therefore, also not the sign of f_o. On the other hand, the transposition of any real root will change the sign of f_o. In this case,

$$f_o \rightarrow -f_o \quad \text{and consequently} \quad z_{in}(0) \rightarrow \frac{1}{z_{in}(0)} \qquad (VI.19)$$

One may verify this in the table of Fig. VI.3. This choice to change the termination to its reciprocal does not exist for filters having no real roots, for instance antimetrical lowpass filters.

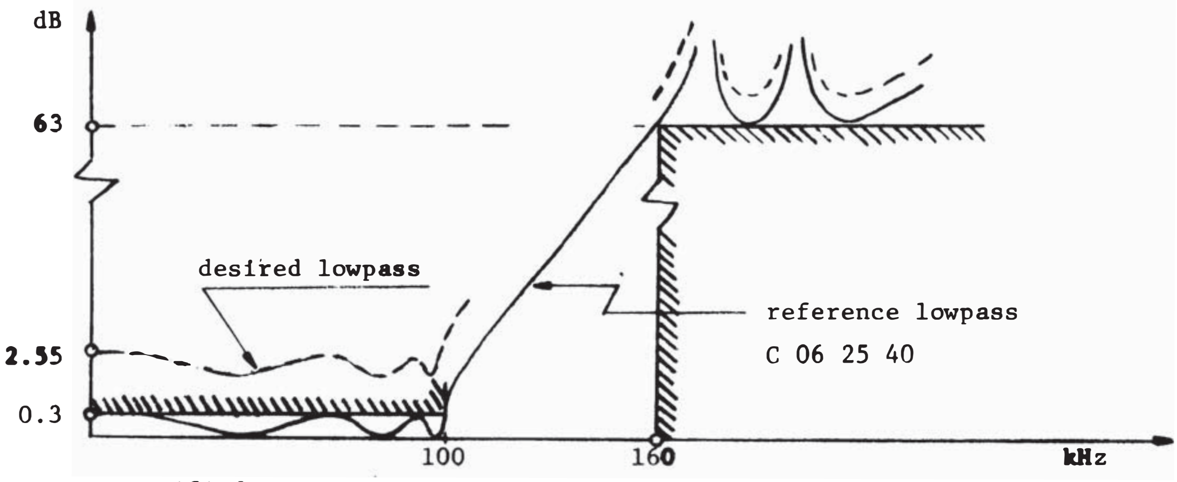
By means of the computer program SYNTH, the design of lowpass filters with arbitrary terminations can be carried out either from H(s) or from K(s).

Example for an H(s) design

Design a Cauer parameter lowpass of 6th degree with

passband	limit = 100 kHz	ripple $\leq$ 0.3 dB
stopband	limit = 160 kHz	discrim $A_{min} \geq$ 60 dB
terminations	R_{gen} = 60 Ohms	R_{load} = 300 Ohms

According to [CE-1], a lowpass C 06 25 40 can be used as a reference filter.



For the specified terminations,

(VI.1) $\rightarrow$ $A_o = 2.55$ dB

and therefore, the necessary shift $\Delta A = 2.55 - 0.3 = 2.25$ dB.

In the computer program, use the tabulated parameters of [CE-1], pg. 139, and specify 2.25 dB at the absolute minimum of H(s).

Data Cards for complete Synthesis

```
COLUMN   1    1    2    2    3    3    4    4    5    5    6    6    7    7    8
........0....5....0....5....0....5....0....5....0....5....0....5....0....5....0

 COMPLETE SYNTHESIS
 LOWPASS C 06 25 40 WITH FLAT LOSS                                  H(S)-DESIGN
 DEGREE=  6 NATURAL MODES        0 REAL              3 PAIRS
             ATTEN.POLES         0 AT THE ORIGIN     2 PAIRS          0 QUADS
 REF.FRQ. 100.0    KHZ,FLAT L. 2.25     DB
 EVAL.    LOSS RESPONSE          LOSS RESPONSE
 FROM      0.0      KHZ           100.0    KHZ
 TO        100.0    KHZ           150.0    KHZ
 SCALE    LINEAR                 LINEAR
 WITH      2.0      KHZ INCR.     1.0      KHZ INCR.
 PLOT MARGINS
 LEFT      2.0      DB            0.0      DB
 RIGHT     5.0      DB            100.0    DB
 SUBDIV.   0.25     DB            20.00    DB
 NAT.MODES    RE(KHZ)                IM(KHS)
 A(1)=     -7.025408      B(1)=  102.074788
 A(2)=     -7.025408      B(2)=  80.082854
 A(3)=     -7.025408      B(3)=  31.591167
 ATTEN.PLS.  RE(KHZ)                 IM(KHZ)
 X(1)=      0.0           Y(1)=  164.481
 X(2)=      0.0           Y(2)=220.304
 REALIZATION DATA
 GEN.IMP. 60.0               OHM
   1 REALIZATION
D  2   1   3   4

COLUMN   1    1    2    2    3    3    4    4    5    5    6    6    7    7    8
........0....5....0....5....0....5....0....5....0....5....0....5....0....5....0
```

The computer will first determine the absolute minimum of H(s), which must occur at one of the reflection zeroes of the reference filter. It will then calculate the constant of H(s) such that the minimum loss is 2.25 dB. Subsequently, it determines the reflection zeroes which correspond to this new H(s) and selects those in the left half plane. The program proceeds then in the previously described manner. For another distribution of the reflection zeroes, a second run is necessary.

Example for a K(s) design

Design a Cauer parameter lowpass of 7th degree with

passband	limit = 100 kHz	ripple	0.1 dB
stopband	limit = 115 kHz	discrim.	40 dB
terminations	R_{gen} = 60 Ohms	R_{load}	= 300 Ohms

In this example, a lowpass C 07 15 60 was selected from [CE-1], pg. 183, as reference lowpass having a constant C_1 = 23.07. Therefore r = 300 / 60 = 5

$$\text{(VI.9)} \rightarrow \gamma = 1.3416 \quad ; \quad \text{(VI.13)} \rightarrow C_2 = 34.6$$

Within the same parameter block on page 183 one may find a row with "C = 34.8" belonging to a reference filter with 15% reflection coefficient. According to (VI.15), its natural modes or their counterparts in the right half plane are the reflection zeroes of the desired lowpass. With these the following data cards may be prepared:

Data cards for a complete synthesis

```
COLUMN   1    1    2    2    3    3    4    4    5    5    6    6    7    7    8
.........0....5....0....5....0....5....0....5....0....5....0....5....0....5....0

  COMPLETE SYNTHESIS
  LOWPASS C 07 15 60 WITH FLAT LOSS                                        K(S)-DESIGN
  DEGREE    7  REFL.ZEROES       0 AT THE ORIGIN      3 PAIRS               1 REAL
               ATTEN.POLES       0 AT THE ORIGIN      3 PAIRS               0 QUADS
  REF.FRQ. 100.0    KHZ     A0=  2.55     DB  AT      0.0      KHZ
  EVAL.    LOSS REPONSE         LOSS RESPONSE
  FROM      0.0     KHZ          100.0    KHZ
  TO        100.0   KHZ          150.0    KHZ
  SCALE    LINEAR               LINEAR
  WITH      2.0     KHZ INCR.    1.0      KHZ INCR.
  PLOT MARGINS
  LEFT      2.0     DB           0.0      DB
  RIGHT     4.0     DB           80.0     DB
  SUBDIV.   0.5     DB           20.0     DB
  REFL.ZER.   RE(KHZ)               IM(KHZ)
  A(0)=     -54.85927     B(0)=  0.0
  A(1)=     +37.36940     B(1)=  65.60974
  A(2)=     -16.98871     B(2)=  94.01670
  A(3)=     +3.61325      B(3)=  102.228525
  ATTEN.PLS.  RE(KHZ)               IM(KHZ)
  X(1)=     0.0           Y(1)=  116.887
  X(2)=     0.0           Y(2)=  132.353
  X(3)=     0.0           Y(3)=  207.756
  REALIZATION DATA
  GEN.IMP.       60.0         OHM
    1 REALIZATION
D  3   1   2   4
.
COLUMN   1    1    2    2    3    3    4    4    5    5    6    6    7    7    8
.........0....5....0....5....0....5....0....5....0....5....0....5....0....5....0
```

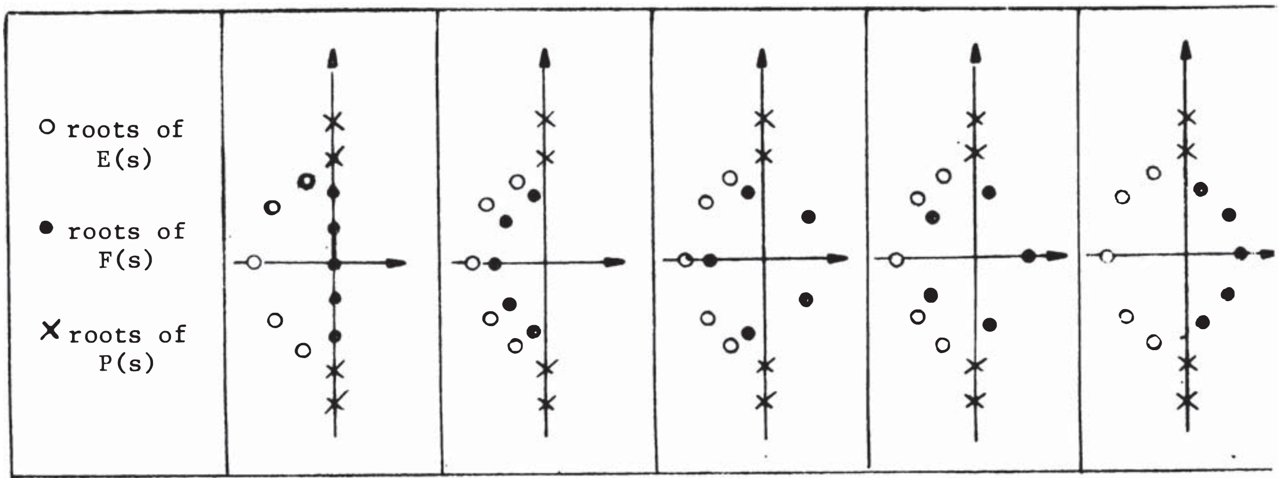
The method has the following advantages:

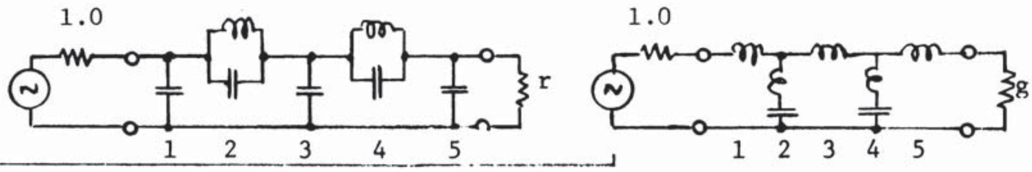
(a) The reflection zeroes can be distributed arbitrarily between the left and right half plane.

(b) The K(s) mode in SYNTH1 permits filters with a higher degree.

(c) Only once need the program find the roots of a polynomial, normally the most time-consuming process.

The method can be used only if two compatible (in the above sense) sets of natural modes are available.





c_1	ℓ_1	0.5654	3.2219	0.5654	0.5654	0.2323
c_2	ℓ_2	0.2150	0.9869	0.1979	0.2320	0.2243
ℓ_2	c_2	0.5167	0.1126	0.5613	0.4787	0.4952
ω_2	ω_2	3.0000	3.0000	3.0000	3.0000	3.0000
c_3	ℓ_3	0.6952	3.2295	1.9200	0.4503	0.6551
c_4	ℓ_4	0.0952	0.4847	0.4381	0.0205	0.0877
ℓ_4	c_4	0.6566	0.1289	0.1426	3.0414	0.7124
ω_4	ω_4	4.0000	4.0000	4.0000	4.0000	4.0000
c_5	ℓ_5	0.6483	1.5882	3.2416	0.1297	0.7205
r	g	1.0000	0.2000	0.2000	5.0000	5.0000

Figure VI.3 5th Degree Low-Pass Filters with Equal and Non-Equal Terminations

For instance, if R_{gen} = 60 Ohms, R_{load} = 90 Ohms, then

$$r = 1.5 \qquad \gamma = 1.02062 \qquad C_2 = 115.352$$

A row with a constant C = 115.352 is not tabulated. However, it may easily be generated by a computer run with the tabulated reflection zeroes and attenuation poles of the block and a suitable A_{min}. By extrapolation of the tabulated A_{min}'s of the block vs. the tabulated C's, one may find a suitable A_{min} = 56.4 dB. Specifying this attenuation at the cut-off frequency 1.1547 on data card 4 will yield a set of natural modes, which may be used as reflection zeroes as previously.

H(s) and K(s) of equations VI.11 and 14 yield a transmission network as shown in the top part of Fig. VI.4. However, most synthesis program will not directly calculate the turns ratio of the ideal transformer contained in the network. These programs conventionally print the actual terminations as in Figure VI.3, for instance. These actual operating condition, the objective of "flat loss", are easily derived as indicated in Fig.VI.4.

2. Extreme terminations

Of particular interest are operating conditions where one or the other termination assumes the extreme values 0 or ∞. At the input side, the generator is then either an ideal voltage or current source; at the output the load degenerates to an open or short circuit. In both cases the consideration of transducer loss becomes meaningless because

$P_{ref} = \infty$ for an ideal source, or

$P_{load} = 0$ for no resistive termination

Consequently,

(VI.9): $\gamma = \frac{1}{2}(\sqrt{r} + 1/\sqrt{r}) \rightarrow \infty$

(VI.8): $\Delta A = 10 \log_{10} \gamma^2 \rightarrow \infty$

(VI.13): $C_2 \rightarrow C_1$

Of special significance is that

(VI.15): $F(s)\,F(-s) \rightarrow F_1(s)\,F_1(-s) + C_1^{-2}\,P_1(s)\,P_1(-s) = E_1(s)\,E_1(-s)$

and therefore,

$$F(s)\ F(-s) = E_1(s)\ E_1(-s) \tag{VI.24}$$

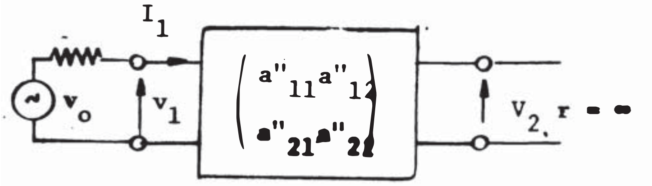
$$F(s) = \pm E_1(s) \tag{VI.25}$$

$$\text{or} \quad F(s) = \pm E_1(-s) \tag{VI.26}$$

As the transducer loss increases, the reflection zeroes move closer towards the roots of $E_1(s)\ E_1(-s)$; in the limiting case of extreme terminations the reflection zeroes coincide with the natural modes or their counter parts in the right half plane.

The transfer properties and the design equations for these special terminations are easily derived from the equations (VI.20) to (VI.23) in Fig. VI.4. For instance,

Open-circuit output terminals



$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} a''_{11} & a''_{12} \\ a''_{21} & a''_{22} \end{pmatrix} \begin{pmatrix} V_2 \\ 0 \end{pmatrix} \tag{VI.27}$$

$$V_o = V_1 + 1 \cdot I_1 = (a''_{11} + a''_{21})\ V_2 \tag{VI.28}$$

(a) $P_1(s)$ = even [equations (VI.22) of Fig.VI.4]

In order for z_{11} to be meaningful

$$F_e = -E_{1e},\ F_o = +E_{1o} \quad \rightarrow \quad F(s) = -E_1(-s) \tag{VI.29}$$

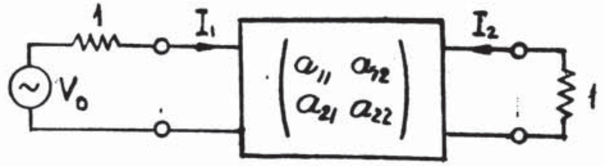
(VI.28):
$$V_o = C_1 \left[\frac{E_{1e}}{P_1} + \frac{E_{1o}}{P_1}\right] V_2 \quad \rightarrow \quad \frac{V_o}{V_2} = H_1(s) \tag{VI.30}$$

V_2 is 6 dB higher in comparison with V_2 of the equally terminated network according to equation (IV.6).

The network can be realized from the following z-parameters:

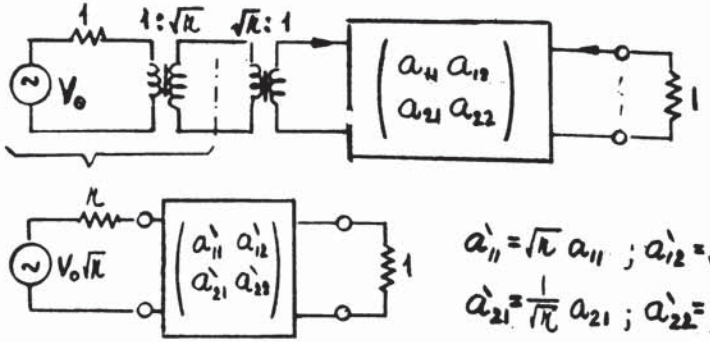
$$z_{11} = \frac{a''_{11}}{a''_{21}} = \frac{E_{1e}}{E_{1o}}\ ; \quad z_{12} = \frac{1}{a''_{21}} = \frac{P_1(s)\ C_1^{-1}}{E_{1o}} \tag{VI.31}$$

Because of no output current, the third parameter $z_{22} = \frac{a''_{22}}{a''_{21}}$ assumes an indeterminate form ($\infty \cdot 0$) which makes z_{22} arbitrary. For economy of elements, in most cases $z_{22} = z_{12}$.



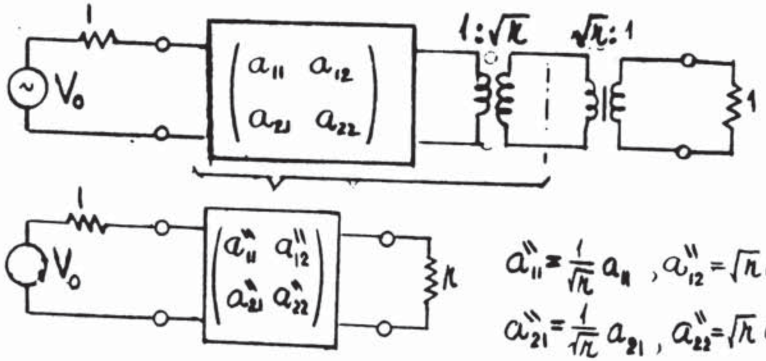
$$a_{11} = \gamma C_1 \left(\frac{E_1 - F}{P_1} \right)_e \; ; \quad a_{12} = \gamma C_1 \left(\frac{E_1 - F}{P_1} \right)_o$$

$$a_{21} = \gamma C_1 \left(\frac{E_1 + F}{P_1} \right)_o \; ; \quad a_{22} = \gamma C_1 \left(\frac{E_1 + F}{P_1} \right)_e$$



$$a'_{11} = \sqrt{n}\, a_{11} \; ; \quad a'_{12} = \sqrt{n}\, a_{12}$$

$$a'_{21} = \frac{1}{\sqrt{n}}\, a_{21} \; ; \quad a'_{22} = \frac{1}{\sqrt{n}}\, a_{22}$$



$$a''_{11} = \frac{1}{\sqrt{n}}\, a_{11} \; , \quad a''_{12} = \sqrt{n}$$

$$a''_{21} = \frac{1}{\sqrt{n}}\, a_{21} \; , \quad a''_{22} = \sqrt{n}$$

	$P_1(s) = \text{even}$	$P_1(s) = \text{odd}$
a'_{11}	$= \frac{n+1}{2} C_1 \frac{E_{1e} - F_e}{P_1}$	$= \frac{n+1}{2} C_1 \frac{E_{1o} - F_o}{P_1}$
a'_{12}	$= \frac{n+1}{2} C_1 \frac{E_{1o} - F_o}{P_1}$	$= \frac{n+1}{2} C_1 \frac{E_{1e} - F_e}{P_1}$
a'_{21}	$= \frac{n+1}{2n} C_1 \frac{E_{1o} + F_o}{P_1}$	$= \frac{n+1}{2n} C_1 \frac{E_{1e} + F_e}{P_1}$
a'_{22}	$= \frac{n+1}{2n} C_1 \frac{E_{1e} + F_e}{P_1}$	$= \frac{n+1}{2n} C_1 \frac{E_{1o} + F_o}{P_1}$
z'_{22}	$= \frac{E_{1e} + F_e}{E_{1o} + F_o}$ (VI.20)	$= \frac{E_{1o} + F_o}{E_{1e} + F_e}$ (VI.21)
y'_{22}	$= \frac{E_{1e} - F_e}{E_{1o} - F_o}$	$= \frac{E_{1o} - F_o}{E_{1e} - F_e}$

	$P_1(s) = \text{even}$	$P_1(s) = \text{odd}$
a''_{11}	$= \frac{n+1}{2n} C_1 \frac{E_{1e} - F_e}{P_1}$	$= \frac{n+1}{2n} C_1 \frac{E_{1o} - F_o}{P_1}$
a''_{12}	$= \frac{n+1}{2} C_1 \frac{E_{1o} - F_o}{P_1}$	$= \frac{n+1}{2} C_1 \frac{E_{1e} - F_e}{P_1}$
a''_{21}	$= \frac{n+1}{2n} C_1 \frac{E_{1o} + F_o}{P_1}$	$= \frac{n+1}{2n} C_1 \frac{E_{1e} + F_e}{P_1}$
a''_{22}	$= \frac{n+1}{2} C_1 \frac{E_{1e} + F_e}{P_1}$	$= \frac{n+1}{2} C_1\, E_{1o} + F_o$
z''_{11}	$= \frac{E_{1e} - F_e}{E_{1o} + F_o}$ (VI.22)	$= \frac{E_{1o} - F_o}{E_{1e} + F_e}$ (VI.23)
y''_{11}	$= \frac{E_{1e} + F_e}{E_{1o} - F_o}$	$= \frac{E_{1o} + F_o}{E_{1e} - F_e}$

(b) <u>$P_1(s)$ = odd [equations (VI.23) of Figure VI.4]</u>

In similar manner one obtains

$$F_e = +E_{1e},\ F_o = E_{1o} \quad \rightarrow \quad F(s) = +E(-s) \tag{VI.32}$$

$$V_o = C_1 \left[\frac{E_{1o}}{P_1} + \frac{E_{1e}}{P_1}\right] \quad \rightarrow \quad \frac{V_o}{V_2} = H_1(s) \tag{VI.33}$$

The network can be realized from the following z-parameters:

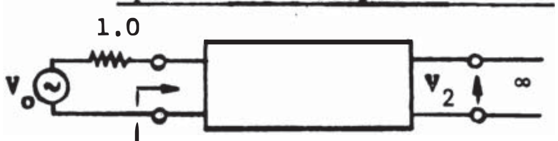
$$z_{11} = \frac{a''_{11}}{a''_{21}} = \frac{E_{1o}}{E_{1e}}\ ; \qquad z_{12} = \frac{P_1(s)\ C_1^{-1}}{E_{1e}} \tag{VI.34}$$

In similar manner, the other operating conditions can also be established. All possible cases are compiled in the table of Fig. VI.5. The rows of this table contain the equations for various generators, the columns for various loads. Th four corner fields are shaded because the networks under consideration must cont at least one resistor.

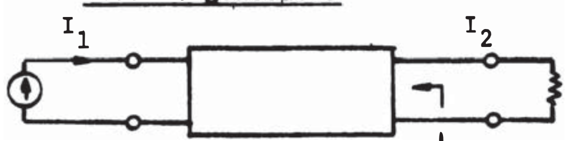
<u>Example</u>: Realize the 5th degree lowpass of the previous subsection for all fou extreme terminating conditions. Its transducer function is

$$H_1(s) = 13.24\ \frac{s^5 + 2.724999\ s^4 + 8.709958\ s^3 + 14.06758\ s^2\ 16.75988\ s + 10.}{(s^2 + 9)\ (s^2 + 16)}$$

<u>Open circuit output terminals</u>

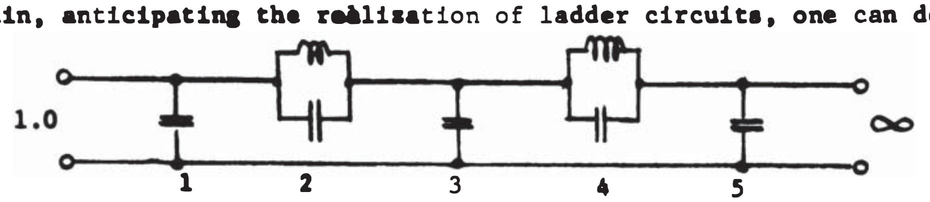


<u>Current generator</u>



$$z_{11} = \frac{E_{1e}}{E_{1o}} = \frac{2.7249\ s^4 + 14.0675\ s^2 + 10.8744}{s^5 + 8.709958\ s^3 + 16.75988\ s} = \frac{E_{1e}}{E_{1o}} = z_{22}$$

Again, anticipating the realization of ladder circuits, one can derive



$c_1 = 0.276$ $\quad c_2 = 0.103$ $\quad c_3 = 0.638$ $\quad c_4 = 0.198$ $\quad c_5 = 0.627$

$\ell_2 = 0.607$ $\quad \ell_4 = 0.560$

$\Omega_2 = 4.0$ $\quad \Omega_4 = 3.0$

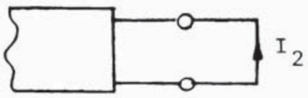
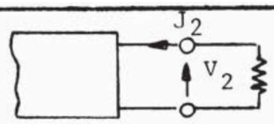
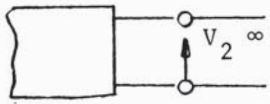
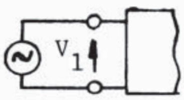
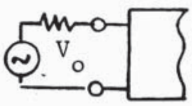
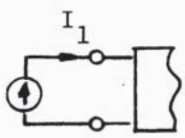
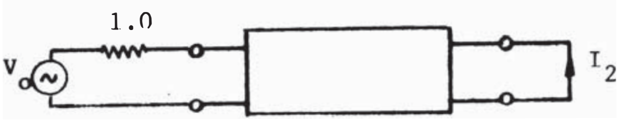
LOAD → SOURCE ↓			
		$H_1(s) = \frac{V_1}{V_2}$ $y_{22} = \frac{E_{1e}}{E_{1o}}$; $P(s)$ = even $y_{22} = \frac{E_{1o}}{E_{1e}}$; $P(s)$ = odd	
	$H_1(s) = \frac{V_o}{-I_2}$ $y_{11} = \frac{E_{1e}}{E_{1o}}$; $P(s)$ = even $y_{11} = \frac{E_{1o}}{E_{1e}}$; $P(s)$ = odd	This field pertains to operating conditions as shown in Section III.	$H_1(s) = \frac{V_o}{V_2}$ $z_{11} = \frac{E_{1e}}{E_{1o}}$; $P_1(s)$=even $z_{11} = \frac{E_{1o}}{E_{1e}}$; $P_1(s)$=odd
		$H_1(s) = \frac{I_1}{-I_2}$ $z_{22} = \frac{E_{1e}}{E_{1o}}$; $P(s)$ = even $z_{22} = \frac{E_{1o}}{E_{1e}}$; $P(s)$ = odd	

Figure VI. 5 Design Equations for Networks with Extreme Terminations

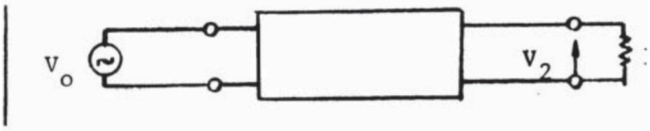
If oriented properly with respect to source and load, the circuit can serve both operating conditions.

Short-circuit output terminals



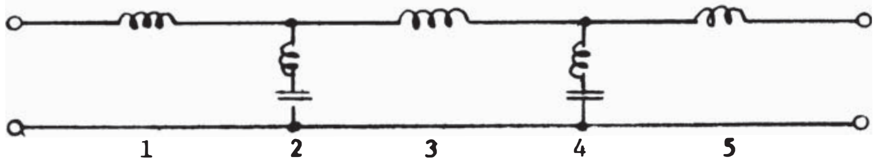
$$H_1(s) = \frac{V_o}{-I_2}$$

Ideal voltage source



$$H_i(s) = \frac{V_o}{V_2}$$

$$y_{11} = \frac{E_{1e}}{E_{1o}} = \frac{2.7249\, s^4 + 14.0675\, s^2 + 10.8744}{s^5 + 8.709958\, s^3 + 16.75988\, s} = \frac{E_{1e}}{E_{1o}} = y_{22}$$



$\ell_1 = 0.276$ $\ell_2 = 0.103$ $\ell_3 = 0.638$ $\ell_4 = 0.198$ $\ell_5 = 0.627$

$c_2 = 0.607$ $c_4 = 0.560$

$\Omega_2 = 4.0$ $\Omega_4 = 3.0$

If oriented properly with respect to source and load, the circuit may again serve both operating conditions. One may also notice the resulting circuits must be dual.

The computer program SYNTH can also aid the design of transmission networks where one of the terminations assumes extreme values. Such designs should be carried out from the coefficients of E(s) and with all coefficients of F(s) identically zero, in the mode "REALIZATION ONLY". The program available as a service from CDC Cybernet has been modified to print the proper terminations 0 or ∞ whatever the case may be. Because the computer produces a dual pair of circuits, only one computer process is necessary to produce all four possible termination cases.

The method and the input data will be demonstrated by an unconventional hybrid realization of the single-sideband bandpass of subsection V.3. By the conventional methods, one may find the following design parameters.

	refl. zer. [kHz]	natural modes [kHz]	atten. pls. [kHz]
1	$\pm j\ 94.744$	$-3.5670874 \pm j\ 91.066$	0 2nd ord.
2	$\pm j\ 98.470$	$-7.3827205 \pm j\ 97.915$	$\pm j\ 119.793$
3	$\pm j\ 103.049$	$-5.5599663 \pm j\ 104.932$	$\pm j\ 131.383$
4	$\pm j\ 105.812$	$-1.8613504 \pm j\ 108.161$	∞ 2nd ord.

From these one may form the overall transducer function in the following form:

$$H(s) = H_1(s) \cdot H_2(s) \quad \text{where}$$

$$H_1(s) = C_1 \frac{s^4 + 0.259647\ s^3 + 2.084933\ s^2 + 0.2710239\ s + 1.064664}{s^2}$$

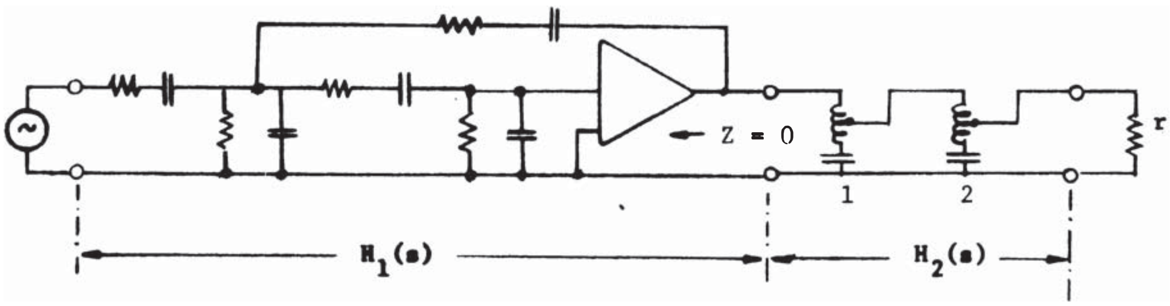
$$H_2(s) = C_2 \frac{s^4 + 0.1085687\ s^3 + 2.003473\ s^2 + 0.114406658\ s + 0.9719685}{(s^2 + 1.19793^2)(s^2 + 1.31383^2)}$$

$H_1(s)$ is formed from the natural modes 2 and 3 which have the largest real parts, $H_2(s)$ of the others. Because of stability and ease of adjustment, one may consider the second transducer function as a passive LC ladder circuit. The first could possibly be an active RC device.

Input data for the computer aided synthesis of $H_2(s)$

```
COLUMN    1    1    2    2    3    3    4    4    5    5    6    6    7    7    8
.........0....5....0....5....0....5....0....5....0....5....0....5....0....5....0
.
 REALIZATION ONLY
 HYBRID REALIZATION
 DEGREE OF E(S)=    4   DEGREE OF F(S)=   4   DEGREE OF P(S)=   2   NO.OF ATT.PLS.=  2
 REF.FREQUENCY            100.0 KHZ             REF.RES.                 1000.0    OHM
  X1=       0.0            Y1=  119.793          KHZ
  X2=       0.0            Y2=  131.383          KHZ
 E0=      0.9719768
 E1=      0.114406658
 E2=      2.003473193
 E3=      0.10856874
 E4=      1.0
 F0=      0.0
 F1=      0.0
 F2=      0.0
 F3=      0.0
 F4=      0.0
  2 REALIZATIONS
 D  1   2
D  2   1
.
COLUMN    1    1    2    2    3    3    4    4    5    5    6    6    7    7    6
.........0....5....0....5....0....5....0....5....0....5....0....5....0....5....0
```

Resulting overall circuit



Element values for a load resistor 1000 Ohms:

L_1 = 0.254 mH, C_1 = 10.6 nF, $f_{\infty 1}$ = 119.793 kHz, tap at 0.654 of total turns

L_2 = 0.981 mH, C_2 = 2.49 nF, $f_{\infty 2}$ = 131.383 kHz, tap at 0.600 of total turns

In attempting a hybrid circuit as above, one should be aware of severe limitations. Firstly, the passbands of the two realizations are both curved, only their superposition generates an equal ripple response, theoretically. In practice, the compensation will not be perfect. Secondly, one loses at least partly the advantagesof an overall LC design. Such a desigh has inherently a zero-sensitivity of the transducer loss at all reflection zeroes. * This advantage is partially lost in the hybrid design. However, with the high quality ferrites and capacitors now available, the attenuation poles will be very stable.

* Orchard, H.T.: "Inductorless Filters: (Electronics Letters, Vol. 2, Pages 224 - 225, June 1966)

VII. Realization Methods

Zie erratum

After having arrived at suitable functions K(s) and H(s) such that a set of specifications can be satisfied, the designer must decide on which network type would be most economical as a realization. To this, many factors must be taken into consideration. The realization by LC networks should be included in these considerations because they are in most cases cheaper and more reliable. In order to decide correctly for or against LC networks, the designer should understand the basic concepts of their design and also the limitations implied by the components. He should be able to visualize one or several potential networks and relate these, qualitatively, to functions K(s) and H(s) of corresponding complexities. He should also be able to relate given functions K(s) and H(s) to several possible circuits. Finally, he should be able to anticipate in his mind the necessary network transformations by which many circuits must be implemented. In these activities, the neophyte and the experienced designer are aided by conventional methods and procedures and by tables. Some of these are the subject of this section.

As in the case of reactive two-terminal networks, one can distinguish between realizations which require a minimum number of circuit elements (canonical configurations) and others which exceed this minimum number. In case of LC transmission networks, these non-canonical structures are often preferred for practical reasons, particularly for ease of tuning, adjustment and stability. One may also distinguish between two-parts which provide only one path along which energy can be transmitted from the input to the output and others with multipath transmission. In multipath structures such as lattices, bridged T's, double T's or parallel ladder circuits, attenuation poles are generated by cancellation of energies arriving at the output terminals along different paths. These cancellations are very sensitive to element variations and the adjustment of only one circuit element usually shifts all attenuation poles. This makes adjustment difficult. An exception to this is the parallel tuned circuit in the series arm of a ladder structure. It provides two paths of energy between two nodal points of the circuit: one through the coil the other through the capacitor. No transmission occurs if these energies have equal amplitude but opposite sign which occurs at the resonant frequency. However, adjustment of this parallel circuit effects only one attenuation pole, so alignment is simple.

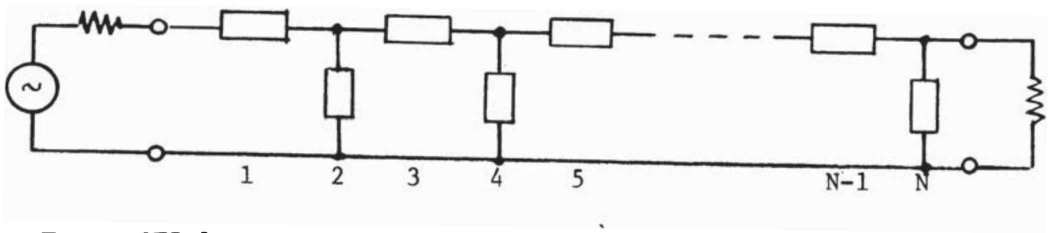
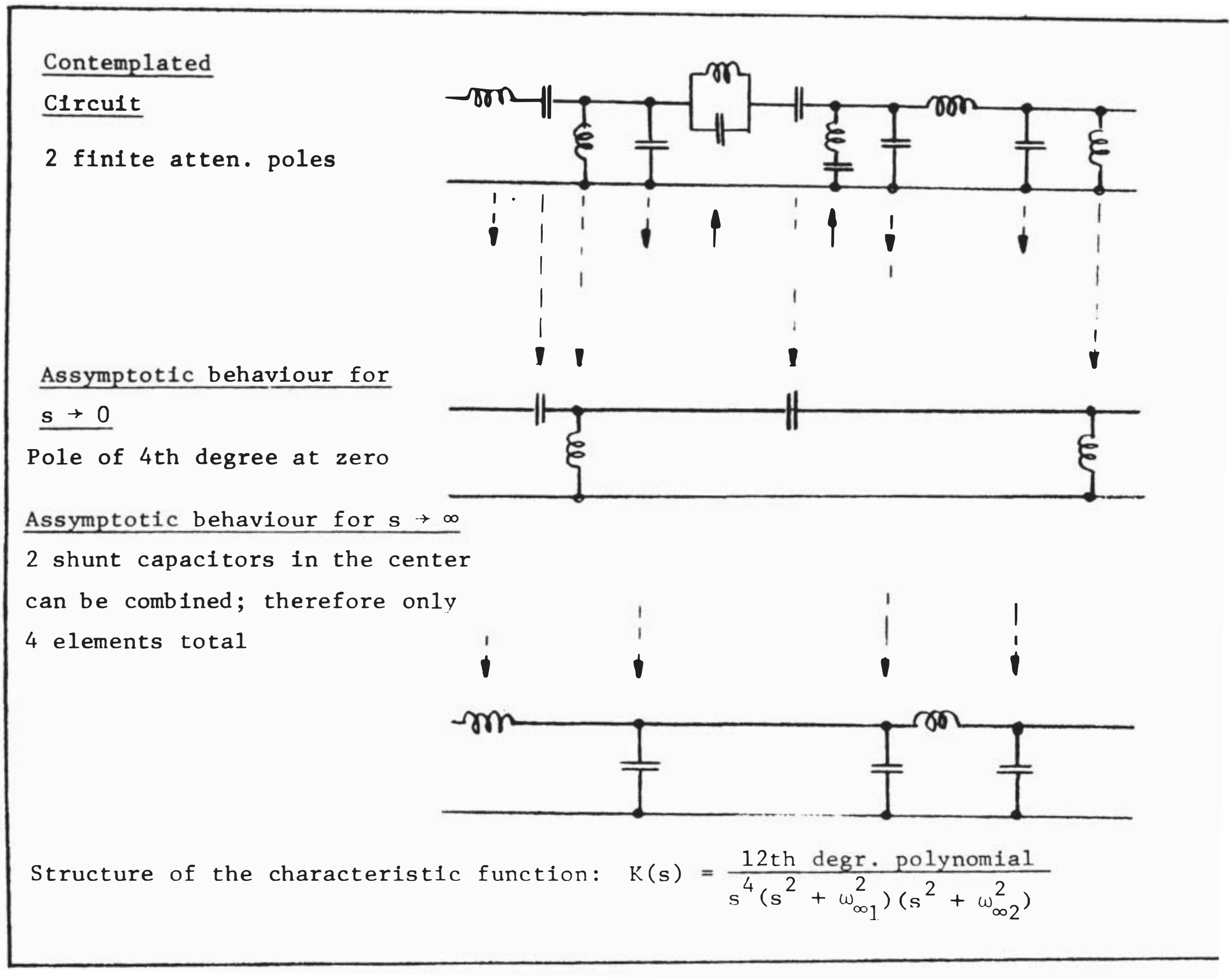


Figure VII.1 Transmission Network with Ladder Structure



$$K(s) = \frac{\text{12th degr. polynomial}}{s^4(s^2+\omega_{\infty 1}^2)(s^2+\omega_{\infty 2}^2)}$$

Figure VII.2 The Derivation of K(s) for a Specified Circuit

Dual considerations apply to series resonant circuits in shunt branches of ladder structures. In spite of these facts, ladder circuits are considered as one-path transmission networks.

In ladder circuits with positive elements, attenuation poles can only occur at the reactance poles of the series branches z_1, z_3, z_5 etc. or at the reactance zeroes of the shunt branches z_2, z_4, z_6 etc. (see Fig. VII.1). However, one must not conclude that the number and type of these reactance poles and zeroes is identical to the number and type of attenuation poles of the network. (For a proof of this, see[BA-1], pgs. 151-153.) In order to relate a given ladder circuit to a particular characteristic function, the following rule could be followed although it does not apply in all cases. (See Fig. VII.2):

(a) Count the number of resonant circuits which block the signal flow. Each of these corresponds to a factor $(s^2 + \omega_{\infty\nu}^2)$ in the denominator of K(s). In Fig. VII.2, there are two such circuits which contribute $(s^2 + \omega_{\infty 1}^2)$ $(s^2 + \omega_{\infty 2}^2)$ to P(s).

(b) Redraw the circuit according to its asymptotic behaviour as $s \to 0$. This eliminates all capacitors in shunt branches and all inductors in series branches. Combine elements, if possible. The number "m" of remaining elements is equal to the multiplicity of the attenuation pole at 0. In Fig. VII.2, m = 4.

(c) Redraw the original circuit according to its asymptotic behaviour as $s \to \infty$. Eliminate and combine elements in an analogous manner. The number of remaining elements is equal to the multiplicity of the attenuation pole at infinity. It is also equal to the number by which the degree of F(s) exceeds the degree of P(s). In Fig. VII.2, this yields a numerator of 12th degree. Its 12 reflection zeroes can be chosen according to the specifications, for instance to make the passband maximally flat, or equal ripple or to give it any other desired shape.

Most experienced designers proceed in the manner that they first contemplate a circuit which may satisfy the requirements; then they determine the complexity of the related K(s) and carry out the approximation procedure with this function. Finally, they control the realization procedure such that they arrive at the numerical values of the contemplated circuit.

1. Canonical ladder realizations

For symmetrical and antimetrical networks, there exist two canonical realizations which are of limited practical interest. These are the lattice and the partial fraction realization. For these, reference is made to various textbooks

on synthesis. (For instance [GU-1], pgs. 194-288; [VA-1], pgs. 313-367; [CA-1], pgs. 439-446.) The discussion will be restricted to canonical ladder configurations because they do have some practical importance and they are the basis for the most commonly used non-canonical ladder configurations.

Procedures leading to ladder configurations in general are encountered in the realization of arbitrary impedances according to Brune's method ([GU-1], pgs. 343-356; [VA-1], pgs. 161-172). According to this method, it is first necessary to remove from the driving point impedance or admittance all j-axis poles at finite or infinite frequencies.

Alternating between these "reactance-susceptance reductions", one may succeed in reducing the impedance to a constant term, namely a resistor. The reactive components of this reduction form a lossless transmission network and the last resistor is the load.

However, if all possible reactance-susceptance reductions are exhausted without having achieved a complete reduction then the ladder development can be continued by one or several consecutive Brune cycles. The normalized impedance at this stage of the reduction is then necessarily of the form:

$$z(s) = \frac{n(s)}{d(s)} = \frac{\sum_{v=o}^{n} n_v s^v}{\sum_{v=o}^{n} d_v s^v} \qquad \text{(VII.1)}$$

where all coefficients are > 0. Performed in the conventional manner, each Brune section will contain one resistor and three reactive elements. Details may be found in most textbooks on network synthesis. As a result of this procedure a ladder structure similar to the one shown in Fig. VII.3 may be expected. If the perfectly coupled coils of a Brune network are counted as one element, namely as a tapped coil, then each block in Fig. VII.3 reduces the rank of the impedance by the same number as there are reactive elements in the block. It is significant for this type of realization of an impedance:

(a) that the resulting network will contain resistors throughout the network and, therefore, power delivered to the input terminal will be dissipated in all resistors and not solelv in the load as desired.

(b) that the sequence of Bruen sections is not arbitrary,but depends on the real part of the driving point impedance at each stage of the removal.

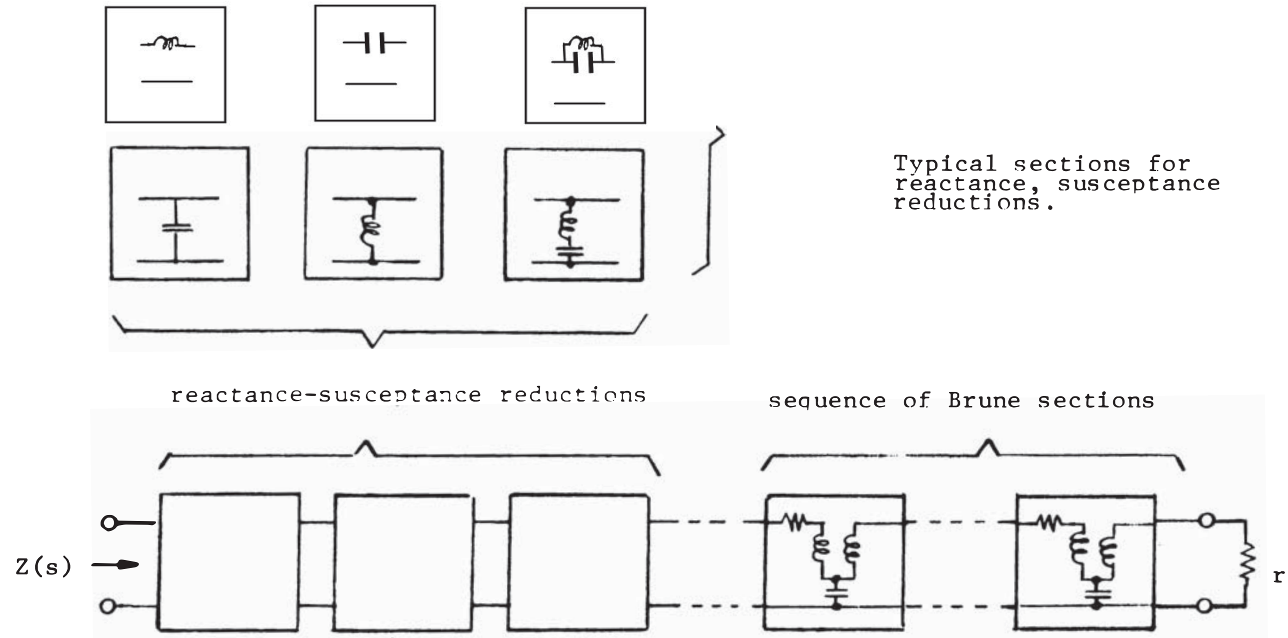


Figure VII. 3 Typical Impedance Realization by Brune's Method

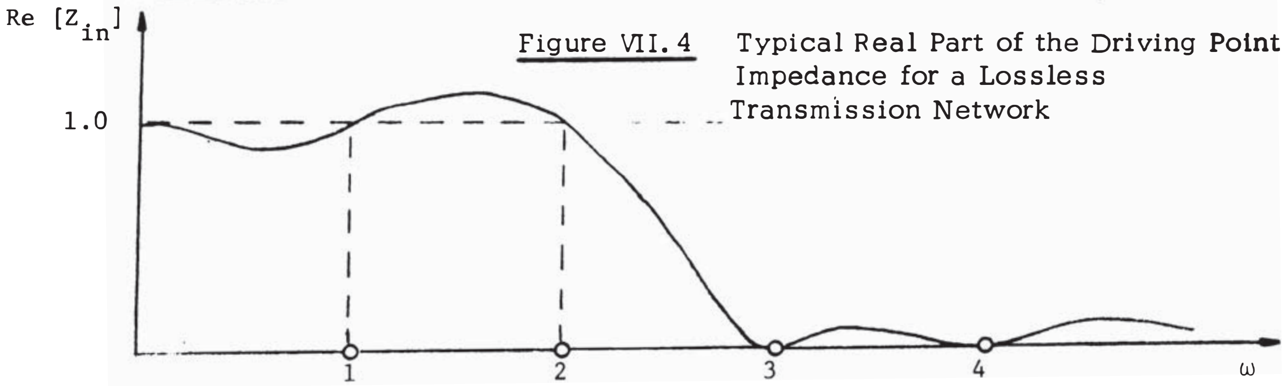


Figure VII. 4 Typical Real Part of the Driving Point Impedance for a Lossless Transmission Network

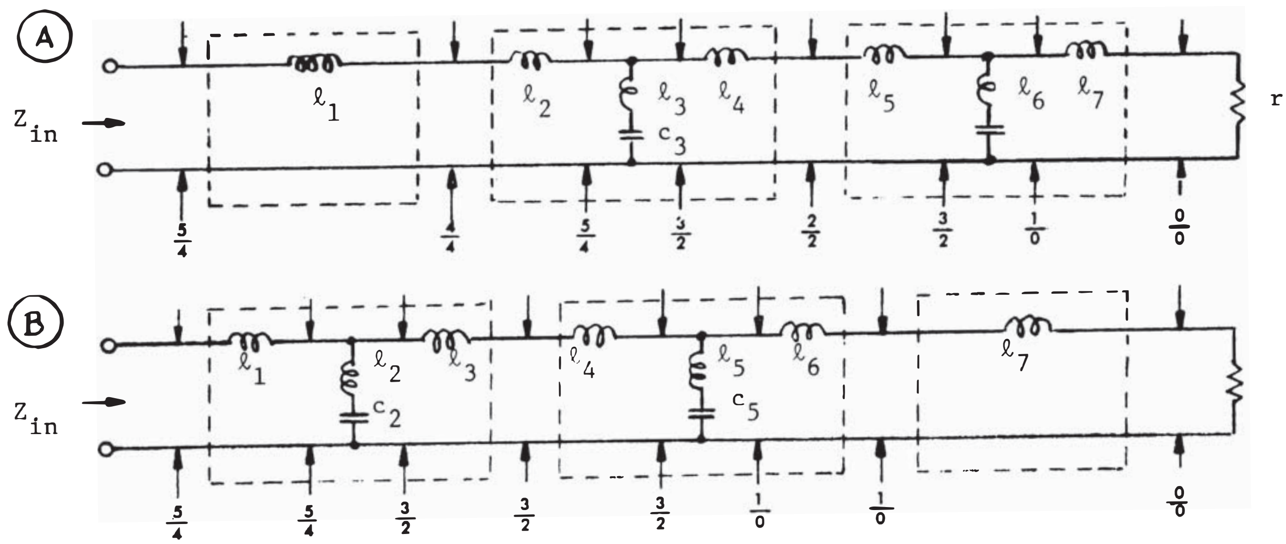


Figure VII. 5 Realization of a 5th Degree Low-Pass by Brune's Method

Driving point impedances of lossless transmission networks are special cases According to equation II.10 and also II.13, one may derive for the real part of $z_{in}(j\omega)$

$$\mathrm{Re}\ [z_{in}(j\omega)] = \frac{1}{2}\ [z_{in}(s) + z_{in}(-s)]_{s=j\omega} = \frac{P(s)\ P(-s)}{[E(s) + F(s)][E(-s + F(-s)]_{s=j\omega}} \quad \text{(VII.2}$$

Consequently, $\mathrm{Re}[z_{in}(J\omega)] = 0$ at all attenuation poles and also at infinity if the degree of E(s) exceeds the degree of P(s). For instance, for the 5th degree lowpass of subsection VI.1 with

$$E_1(s) = C_1[s^5 + 2.724999s^4 + 8.709958s^3 + 14.06758s^2 + 16.75988s + 10.8744]$$

$$F_1(s) = C_1[s^5 \qquad + 5.0s^3 \qquad + 4.0s \qquad]$$

$$z_{in} = \frac{2s^5 + 2.724999s^4 + 13.70996s^3 + 14.06758s^2 + 20.75988s + 10.8744}{s.724999s^4 + 3.709958s^3 + 14.06758s^2 + 12.75988s + 10.8744}$$

one may qualitatively expect a real part as shown in Fig. VII.4. Following the method as illustrated in Fig. VII.3 yields the circuit "A" of Fig. VII.5. However, because the real parts of the impedance is 0 at $\omega_{\infty 1}$, $\omega_{\infty 2}$ and infinity, one may remove the Brune sections first and remove the pole at infinity last (circuit "B"). In both circuits, the structure of the impedance is indicated at each stage of the removal procedure. For instance, the fraction "5/4" indicates a 5th degree numerator and a 4th degree denominator.

The realization of the transmission network becomes considerably simpler if the termination of the network is removed either by shorting or disconnecting the load resistor. z_{in} becomes then z_{oc} where

$$z_{oc} = z_{11} = \frac{E_e + F_e}{E_o - F_o} = \frac{2.72499s^4 + 14.06758s^2 + 10.8744}{3.709958s^3 + 12.75988s}$$

$$z_{sc} = \frac{1}{y_{11}} = \frac{E_o + F_o}{E_e - F_e} = \frac{20.s^5 + 13.70996s^3 + 20.75988s}{3.72499s^4 + 14.06758s^2 + 10.8744}$$

The task is now to decompose either of these reactances such that one obtains the same structure as if a decomposition of z_{in} were carried out. In this decomposit: each branch reactance is responsible for an attenuation pole; at these, the prese

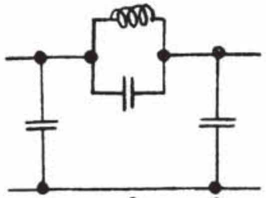
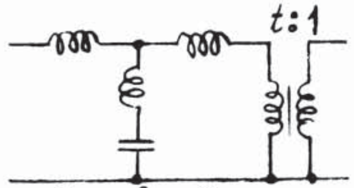
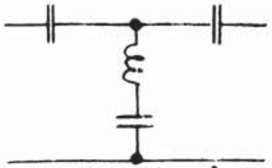
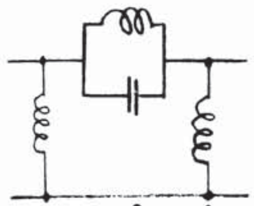
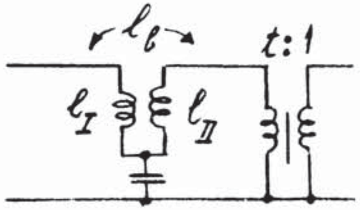
Brune Section	Equival. Condition	Transform. Formulas	Convention. Brune Section
(A) 	$c_1 c_2 + c_1 c_3 + c_2 c_3 = 0$ or $c_3 = -\frac{c_1 c_2}{c_1 + c_2}$	$l_a = \frac{c_3}{c_1 + c_3} l_2$ $l_b = \frac{c_2}{c_1 + c_3} l_2$ $l_c = \frac{c_1}{c_1 + c_3} l_2$ $c_b = (c_1 + c_3$ $t = 1$	$l_a l_b + l_a l_c + l_a l_c =$ or $l_3 = -\frac{l_1 l_2}{l_1 + l_2}$ 
(B) 	$c_1 + c_2 + c_3 = 0$ or $c_3 = -(c_1 + c_2)$	$t = (1 + \frac{c_2}{c_1}$ $l_a = l_2 (1 - t)$ $l_b = l_2 t$ $l_c = l_2 (t^2 - t)$ $c_b = \frac{c_2}{t}$	
(C) 	$l_1 + l_2 + l_3 = 0$ or $l_3 = -(l_1 + l_2$	$t = (1 + \frac{l_2}{l_1})$ $l_a = c_2 (1 - t)$ $l_b = c_2 t$ $l_c = c_2 (t^2 - t)$ $c_b = \frac{l_1 l_2}{l_1 + l_2}$	 $l_I = l_a + l_b$ $l_{II} = l_b + l_c$

Figure VII.6 Brune Sections of First Order

or the absence of the load resistor is irrelevant. For this reason, one can expect the numerical values of the elements to be the same regardless of whether they were obtained from the decomposition of the reactance or the impedance.

Removal of an element is conventionally called the "full or partial removal of an attenuation pole". For instance in circuit "A", ℓ_1 represents the full removal of a pole at infinity; significant is that the rank of the impedance (or the reactance) is lowered by 1. In circuit "B": ℓ_1 represents the partial removal of an attenuation pole at infinity; significant is that the rank of the impedance remains unchanged.

In canonical ladder circuits, attenuation poles at finite frequencies must be removed by Brune sections. In the above circuit, these require three inductances which may be called ℓ_a, ℓ_b and ℓ_c. The first two, ℓ_a and ℓ_b, result from partial or full removals. The third, ℓ_c, is calculated from the condition of perfect coupling

$$\ell_a \ell_b + \ell_a \ell_c + \ell_b \ell_c = 0 \rightarrow \ell_c = -\frac{\ell_a \ell_b}{\ell_a + \ell_b} \qquad \text{(VII.3)}$$

Normally, the inductances ℓ_a and ℓ_c in the series branch are responsible or they contribute to an attenuation pole at infinity. However, because of perfect coupling, the section approaches an ideal transformer as $s \rightarrow \infty$.

Attenuation poles at finite frequencies can also be removed in canonical form by one of the three sections in Fig. VII.6. All of these are equivalent to the conventional form of the Brune section in the right-most column. Because each of these removes one pair of attenuation poles, they could be called "Brune sections of first order". Obviously, the concept can be extended to the removal of an arbitrary number of finite attenuation poles by "Brune sections of higher order". Cauer's partial fraction circuit, for instance, is such a circuit. However, such sections are of little practical value. An exception is the removal of two pairs of attenuation poles of a pole quadruplet. For these, canonical sections are quite useful Canonical ladder circuits were first published by Piloty [PI-1] . In this publication, the removal of sections is carried out with mathematical rigor by means of the decomposition of the chain matrix. An excellent tutorial treatment based on the decomposition of the input impedance may be found in [TU-1] , chapter 9. The analytical details of the removal are discussed briefly in Appendix F.

Zie erratum

A final word on practical aspects. Brune sections do have some practical importance in the field of circuit design. They are occasionally indispensable if negative elements appear in non-canonical ladder circuits. They are always indispensable for the realization of attenuation poles on the real axis. Finally, they are in some cases the most economical solution, not only because they may save one or several elements. They should be considered at least at some stages in an otherwise non-canonical ladder circuit. If the physical structure of the tapped coil can offer a low leakage then these sections work quite well.

2. Non-canonical ladder realizations

The main disadvantage of canonical ladder circuits is the appearance of negative elements. These require inductances with very tight and critical coupling. In many applications, such inductances are impractical, undesirable or impossible. By necessity, negative elements will always appear when a finite attenuation pole is removed by one of the Brune sections in Fib. VII.6. Therefore, it will only be necessary to consider the modification of those sections. The circuit "B" in Fig. VII.5 offers the solution. Its significant difference to circuit "A" is that the attenuation pole at infinity is not removed at the beginning. Consequently, ℓ_1 is potentially positive and its removal from z_{in} does not violate any decomposition rules for one-ports; neither does the removal of the resonance circuit $\ell_2\ c_2$. The reduced impedance is therefore, still realizable. There is no need to complete the Brune section and, in doing so, introduce a negative element. One may rather repeat the procedure for the second attenuation pole and remove, finally, the attenuation pole at infinity. Fig. VII.7 shows the resulting circuit and indicates also the removal steps. By comparison with circuit "B", one may conclude that

$$\ell_I = \ell_1;\ \ell_{II} = \ell_2;\ell_{III} = \ell_3 + \ell_4;\ \ell_{IV} = \ell_5;\ \ell_V = \ell_6 + \ell_7$$

Potentially, all circuit elements are positive; however, the circuit requires now 7 elements rather than 5 as previously.

These introductory remarks are necessary to prepare and justify the rules of the well known "zero shifting method", ([GU-1], pgs. 231-245; [VA-1], pgs. 112-136). Originally, this method was introduced by Bader ([BA-1]).

In complete analogy to canonical circuits, the zero-shifting method decomposes a suitable input reactance. Its objective is to achieve again a ladder

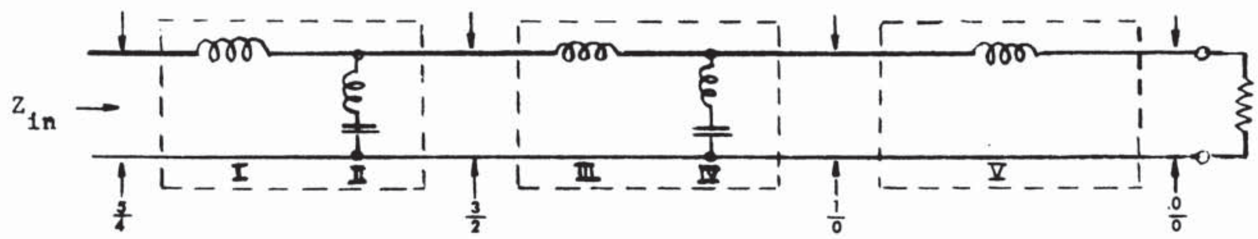


Figure VII. 7 The Non-Canonical Realization of a 5th Degree Low-Pass

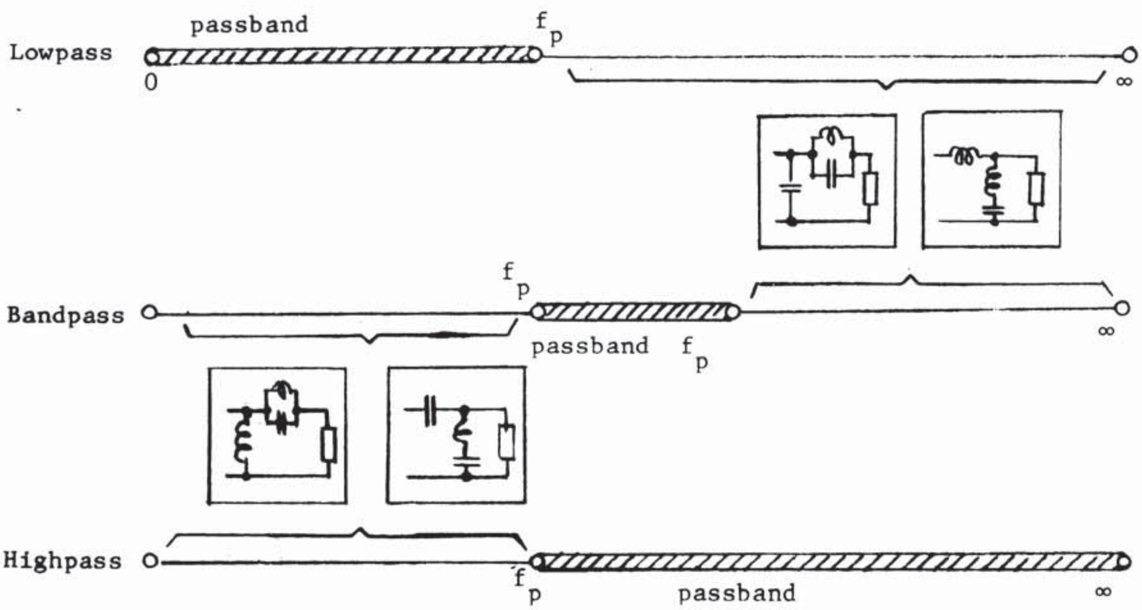


Figure VII. 8 Non-Canonical L-Sections for the Removal of Attenuation Poles

network which could also be obtained by the decomposition of the input impedance. The additional benefit is the control of the decomposition procedure so that the resulting circuit is potentially free of negative elements. To this end, the following two rules must not be violated:

(a) In preparing the full removal of an attenuation pole by a resonance circuit, the partial removal of a reactive branch must be consistent with the structure of the reactance at the point of removal. For instance, an inductance in the series branch can only be removed if the structure of the reactance is either [(2m+1)th degree]/[(2m)th degree] or [(2mth degree]/[2m-1)th degree].

(b) The reactive element which prepares the full removal must not introduce an attenuation pole unless this pole is still present in the remaining network. For instance, the removal of a series inductance is only permitted if the remaining network still contains an attenuation pole at infinity.

Conventionally,the removal of an attenuation pole is carried out by the L-sections shown in Fig. VII.8. One may notice that these sections consist of the left two branches of the Brune sections in Fig. VII.6. The analytical procedure to calculate the element values of these L-sections is identical to the corresponding Brune sections except that the last is not calculated and its removal is not carried out. To facilitate the selection of removal sections without violating rule (a) above, the table of Fig. VII.9 may be consulted. The top line in this table indicates the degree of the "resceptance" at the point of removal. (The word "resceptance" is a convenient contraction of the words "reactance" and "susceptance", similar to the "immittance".) Underneath the top line, shaded and unshaded areas are arranged in horizontal rows. On either side of these rows, the potential structures are indicated and also the type of pole they normally remove. The block at the intersection of a selected "degree" and a selected "type" are shaded if these are imcompatible. Otherwise the block contains the structure of the reduced resceptance after the removal. The circled numbers in the centers of the block quadruplets refer to the particular subroutines in the computer program SYNTH2 which carry out the removals.

The first four rows of the selection table are complete removals of attenuation poles at zero and infinity. The following four rows pertain to procedures by which attenuation poles at finite frequencies are removed. For attenuation poles above and below the passband, the conventional selection of sections is

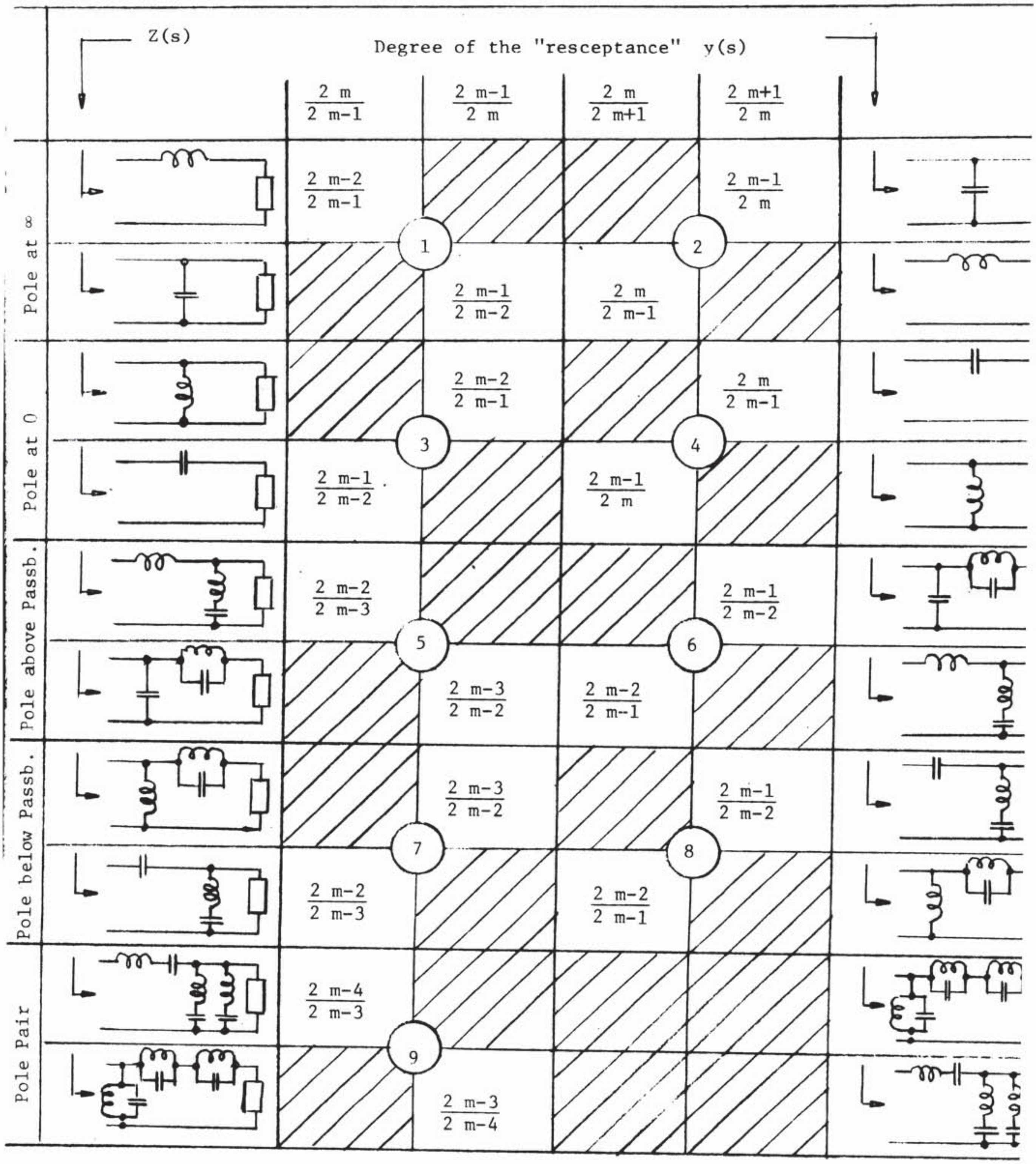


Figure VII.9 Table for the Selection of Removal Sections

Characteristic Function : $K(s = C_k \cdot \dfrac{s^8 + f_6 s^6 + f_4 s^4 + f_2 s^2 + f_0}{s^3(s^2 + \omega_1^2)(s^2 + \omega_2^2)}$

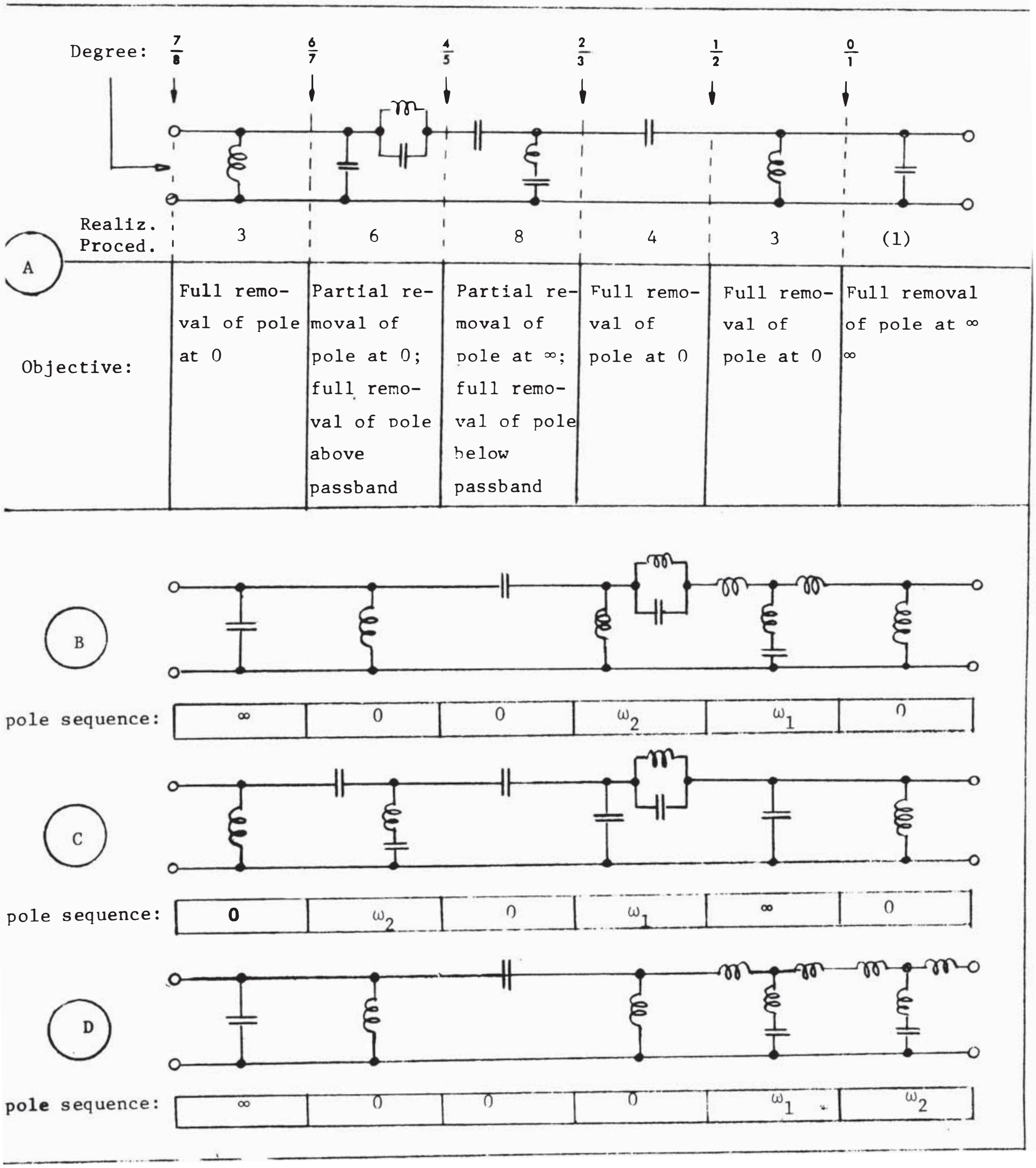


Figure VII. 10 A Demonstrative Example for the Removal of Attenuation Poles

made as shown in Fig. VII.8. The last row pertains to a routine by which two pairs of attenuation poles can be removed simultaneously, one normally above, the other below the passband. This subroutine is also part of SYNTH2.

Fig. VII.10 may serve to demonstrate the use of the selection table. The example is an 8th degree bandpass with the following set of attenuation poles:

$$0 \; 0 \; 0 \; \omega_1 \; \omega_2 \; \infty$$

It is assumed that ω_1 is an attenuation pole above ω_2 below the passband. The decomposition is carried out for the input reactance z_{11} which is of the form 7th degree/8th degree.

The first removal sequence which yields circuit "A" is $0 - \omega_1 - \omega_2 - 0 - 0 -$ The steps of the removal with some explanatory remarks and the structure of the reactance at each stage of the removal are indicated. The other circuits show only the selected sequence of the pole removal and the resulting circuits. Significant for circuit "B" is that its pole removal sequence is the reverse of "A". Comparison of the two circuits shows that a reversed pole sequence does not necessarily yield the network in reverse.

Significant for circuit "C" is the removal of all attenuation poles at 0 and ∞ at the beginning. Consequently, the poles at finite frequencies must be removed by Brune sections. The same circuit would result if the input impedance were decomposed in the conventional way by reactance - susceptance reductions first and Brune sections at the end.

The analytical methods for all removals are similar to those shown in Appendix F. Also, the determination of the turns ratio of the ideal transformer can follow similar concepts. It should also be mentioned that most circuit realizations must be followed by suitable network transformations in order to yield reasonable element values.

3. <u>The computer program SYNTH2 for the realization of ladder networks</u>

In the computer program SYNTH2, the ladder circuits are developed by the decomposition of a suitable input reactance. This may either be z_{oc} or z_{sc} whichever has the higher rank. The circuits derived from these reactances and their dual are printed side by side. Of these, the circuit on the left always uses column 2 or 3 as starting colums in the selection table (Fig. VII.9), depending whether the degree is even or odd.

The decomposition is controlled by the removal sequence of attenuation poles. This sequence is specified by a list of order numbers by which the attenuation poles are stored. The program automatically determines the type of attenuation pole and selects the appropriate subroutine for its removal. Attenuation poles at finite frequencies above the reference frequency will be treated as if they were also above the passband and vice versa. For this reason, it is advisable to select the reference frequency somewhere in the passband range.

The program automatically keeps account of all pole removals regarding the type. Before a transfer is made to a subroutine for the non-canonical removal of an attenuation pole, a check is made if the attenuation pole needed for the partial removal is still available. If not, the removal will be carried out by a Brune section. The designer also has the option of forcing such a removal by making the pertinent order number in the removal sequence negative.

All pole quadruplets are removed by canonical sections. For the physical realization, it will be necessary to perform a network transformation similar to those used with Cauer's partial fraction networks (see, for instance, [VA-1], pgs. 313-318). The computer program also offers the possibility of removing two pairs of attenuation poles simultaneously. In order to assure positive elements, potentially, one of the attenuation poles should be above, the other below the passband. The removal of such a pair is specified by placing an asterisk between the pertinent order numbers. If non-canonical sections are not possible, the removal will be carried out by canonical sections.

4. Consideration of losses

Although the components used for the physical realization of filters has reached a high degree of perfection, they all have at least some inherent losses. Depending on frequency range and application, in many cases the deterioration of the performance due to the losses is negligible or tolerable. If they are not, conventionally one of the following actions are taken:

(a) The addition of an amplitude equalizer (see, for instance, [VA-1], chapter 12). Advantage: the return loss is improved; disadvantage: a flat loss is introduced.

(b) Correction of the resulting circuit preferably by an optimization procedure.

(c) Predistortion of the lossless response to anticipate the distortion.

Of these, the predistortion method will be discussed briefly. In a first approximation, a coil can be modeled by an inductor in series with a resistor; therefore, its impedance is

$$Z_1 = R + j\,\omega\,L = \left(\frac{R}{L} + j\,\omega\right) L = (\sigma_L + j\,\omega)\,L \tag{VII.4}$$

In analogy, the admittance of a lossy capacitor is

$$Y_c = G + j\,\omega\,C = \left(\frac{G}{C} + j\,\omega\right) C = (\sigma_c + j\,\omega)\,C \tag{VII.5}$$

If one assumes $\sigma_L = \sigma_C = \sigma$ for all components of the transmission network, their impedances and admittances will be of the form

$$Z_{1i} = s\,L_i \text{ and } Y_{cj} = s\,C_j \text{ where } s = \sigma + j\,\omega \tag{VII.6}$$

Because all impedances of the transmission network will have the same form as their lossless counterparts, one must be able to derive the identical transducer function as previously. However, to evaluate now the transducer loss one must move along not on the j-axis but at a distance σ to the right in parallel. In SYNTH1, such an evaluation can be performed. In the "K(s)-mode" on data card 4, column 70-77 one may specify $\sigma = f$ [kHz]/Q where f is the frequency in the critical range and Q the quality factor of the components. In practice, only the inductors will introduce losses. To take this into account, one makes the $Q = 2\,Q_1$ where Q_1 is the average Q of all coils.

If the poles and zeroes of H(s) were all shifted to the right by σ, the loss response must be identical to the lossless response. This shift, in principle is the concept of predistortion. In practice, it can only be applied to the natural modes and not to the attenuation poles. If these poles were all at infinity, predistortion is completely correct.

VIII. Z-plane Transformations

In many practical applications, the design of frequency selective networks can be aided by suitable conformal mappings of the original s-plane onto an auxiliary z-plane. Any dependent variable, for instance the transducer loss, will be transposed accordingly.

Several reasons may suggest the use of such transformations. In some applications they serve as an auxiliary step to aid a limited objective in the overall calculations. Such objectives are, for instance to ease the calculation of complex roots or to determine the proper location of attenuation poles, or others. In all these methods the major portion of the calculations is carried back to the s-plane after these limited objectives have been accomplished. Often, as an additional benefit, these methods offer drastically reduced precision requirements which are lost, however, upon return to the s-plane. It is therefore natural to broaden the scope of the original objective by postulating that the entire calculation be performed in the z-plane. Such methods have been proposed by various authors , ([IE-1], [SZ-1], [OT-1]).

Of special interest in the mapping of the s- onto the z-plane is the transposition of the original j-axis because it represents the locus along which the performances are specified. In many applications, it is useful to design the mapping function such that the original passband range is extended to the entire j-axis in the z-plane. For the important case of lowpass filters with a passband range $-j \leq s = j\Omega \leq +j$, this is accomplished by the function

$$z = \frac{s}{\sqrt{s^2 + 1}} \quad ; \qquad s = \frac{z}{\sqrt{1 - z^2}} \qquad \text{(VIII.1)}$$

For tutorial reasons, this rather simple mapping function was selected in preference of others which are more general. These will be mentioned briefly at the end of the section.

1. Polynomial transformation

Let R(s) be a polynomial of n-th degree:

$$R(s) = R_n s^n + R_{n-1} s^{n-1} + \dots + R_1 s + R_o \qquad \text{(VIII.2)}$$

substituting the expression of equation VIII.1 yields:

$$R[s(z)] = R_n \left(\frac{z}{\sqrt{1 - z^2}}\right)^n + R_{n-1} \left(\frac{z}{\sqrt{1 - z^2}}\right)^{n-1} + \dots + R_o$$

After some algebraic manipulations one will arrive eventually at the following form:

$$R[s(z)] = \frac{r_n z^n + r^*_{n-1} z^{n-1} + r_{n-2} z^{n-2} + r^*_{n-3} z^{n-3} + \dots}{(\sqrt{1 - z^2})^n} = \frac{R(z)}{\sqrt{1 - z^2}} \qquad \text{(VIII.3)}$$

In the numerator, the asterisks indicate a factor $\sqrt{1 - z^2}$ which has been omitted. The numerator has the formal appearance of a polynomial and shall therefore, be called a pseudo-polynomial. It can be separated into an even and an odd part

$$R(z) = \begin{cases} = R_e(z) + R^*_o(z) & \text{if } n = \text{even} \\ = R_o(z) + R^*_e(z) & \text{if } n - \text{odd} \end{cases} \qquad \text{(VIII.4)}$$

The asterisks again indicate the attachment of $\sqrt{1 - z^2}$ to all its coefficients.

The transformation of an s-plane polynomial will therefore yield an irrational function with the following properties:

(a) the denominator is the expression $(\sqrt{1 - z^2})^n$;

(b) the numerator is a pseudo-polynomial of the same degree as the original polynomial in the s-plane;

(c) the highest term of this polynomial is always rational;

(d) rational and irrational terms alternate;

(e) when separating the polynomial according to equation (VIII.4), the coefficient in either part depend only on the coefficients of the even and odd part of the original polynomial, respectively. Consequently,

$$R(-z) = R_e(z) - R^*_o(z) \quad ; \quad R(z)\,R(-z) = R^2_e(z) - (1-z^2)\,R^2_o(z) \qquad \text{if } n = \text{even}$$

$$\text{(VIII.5)}$$

$$R(-z) = R_o(z) + R^*_e(z) \quad ; \quad R(z)\,R(-z) = -R^2_o(z) + (1-z^2)\,R^2_e(z) \qquad \text{if } n = \text{odd}$$

Root factors in the s-plane are special types of polynomials. The transformation of these first, second and fourth order polynomials is shown in the table of Fig. VIII.1.

In view of later applications, it is of importance to investigate the decomposition of z-plane polynomials and their separation into root factors which

Location of root	s-plane root factor	z-plane transform
$s_v = u_v$	$(s = d_v)$ with $d_v = -s_v$	$\dfrac{[z + d_v\sqrt{1 - z^2}]}{(\sqrt{1 - z^2})} = \dfrac{[z + d_v^*]}{(\sqrt{1 - z^2})}$
$s_v = \pm u_v$	$(s + d_v)(s - d_v)$	$\dfrac{[z + d_v^*][z - d_v^*]}{(\sqrt{1 - z^2})^2} = \dfrac{[(1 + d_v^2)z^2 - d_v^2]}{(\sqrt{1 - z^2})^2}$
$s_v = u_v \pm jv_v$	$(s^2 + p_v s + q_v)$ with $p_v = - 2u_v$: $q_v = u_v^2 + v_v^2$	$\dfrac{[z^2 + p_v^* z + (1-z^2)q_v}{(\sqrt{1 - z^2})^2} = \dfrac{[a_v z^2 + b_v^* z + c_v]}{(\sqrt{1 - z^2})^2}$ with $a_v = (1 - q_v)$; $b_v = p_v$; $c_v = q_v$
$s_v = \pm(u_v \pm v_v)$	$(s^2 + p_v s + q_v)(s^2 - p_v s + q_v)$ $= [s^4 + (2 q_v - p_v^2)s^2 + q_v^2]$	$\dfrac{[a_v z^2 + b_v^* z + c_v][a_v z^2 - b_v^* z + c_v]}{(\sqrt{1 - z^2})^4}$ $= \dfrac{[A_v z^4 + B_v z^2 + C_v]}{(\sqrt{1 - z^2})^4}$ with $A_v = a_v^2 + b_v^2$; $B_v = 2 a_v - b_v^2$; $C_v = c_v^2$; $A_v + B_v + C_v = 1$

Figure VIII. 1 Transformation of Root Factors

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correspond to root factors in the left and the right half s-plane, respectively. The considerations can be restricted to even $Q(z^2)$ which possess only roots of even multiplicity on the imaginary axis in the z-plane. In this case it is possible to represent $Q(z^2)$ in the following manner:

$$Q(z^2) = R(z)\,R(-z) \tag{VIII.6}$$

$$\text{with } R(z) = C\prod^{n}(z + d_v^*)\prod^{m}(a_\mu z^2 + b_\mu^* z + c_\mu) \tag{VIII.7}$$

$$R(-z) = C\prod^{n}(-z + d_v^*)\prod^{m}(a_\mu z^2 - b_\mu^* z + c_\mu)$$

Therefore,

$$Q(z^2) = C^2(-1)^n \prod^{n}[(1 + d_v^2)\,z^2 - d_v^2] \cdot \prod^{m}[A_\mu z^4 + B_\mu z^2 + C_\mu] \tag{VIII.8}$$

Actually, this is the form in which one must bring the polynomial $Q(z^2)$ before a separation of root factors can be performed. In order to do this, one must first decompose $Q(z^2)$ into conventional root factors in the following form:

$$Q(z^2) = C_q \prod^{n}(z^2 - \delta_v^2)\prod^{m}(z^4 + \beta_\mu z^2 + \gamma_\mu) \tag{VIII.9}$$

(a) <u>decomposition of quadratic factors</u>

$$(z^2 - \delta_v^2) = (1 - \delta_v^2)\left[\frac{1}{(1 - \delta_v^2)} z^2 - \frac{\delta_v^2}{(1 - \delta_v^2)}\right] = (1 - \delta_v^2)[(1 + d_v^2)\,z^2 - d_v^2] \tag{VIII.10}$$

$$\text{with } d_v = \sqrt{\frac{\delta_v^2}{1 - \delta_v^2}} \tag{VIII.11}$$

$$= -[\sqrt{1 - \delta_v^2}\,(z + d_v^*)][\sqrt{1 - \delta_v^2}\,(-z + d_v^*)] \tag{VIII.12}$$

Therefore,

$$\prod^{n}(z^2 - \delta_v^2) = \underbrace{\prod^{n}\sqrt{1 - \delta_v^2}\,(z + d_v^*)}\quad \underbrace{(-1)^n \prod^{n}[\sqrt{1 - \delta_v^2}\,(-z + d_v^*)]} \tag{VIII.13}$$

related to: root factors in the left half plane; root factors in the right half s-plane

(b) <u>decomposition of quartic factors</u>

$$(z^4 + \beta_\mu z^2 + \gamma_\mu) = (1 + \beta_\mu + \gamma_\mu) \left[\frac{1}{1 + \beta_\mu + \gamma_\mu} z^4 + \frac{\beta_\mu}{1 + \beta_\mu + \gamma_\mu} z^2 + \frac{\gamma_\mu}{1 + \beta_\mu + \gamma_\mu}\right] \tag{VIII.14}$$

$$= q_\mu^2 \, [A_\mu z^4 + B_\mu z^2 + C_\mu]$$

with

$$q_\mu = \sqrt{1 + \beta_\mu + \gamma_\mu}; \; A_\mu = \frac{1}{q_\mu^2}; \; B_\mu = \frac{\beta_\mu}{q_\mu^2}; \; C_\mu = \frac{\gamma_\mu}{q_\mu^2}; \tag{VIII.15}$$

Therefore,

$$\prod^{m} (z^4 + \beta_\mu z^2 + \gamma_\mu) = \underbrace{\prod^{m} q_\mu \, (a_\mu z^2 + b^*_\mu z + c_\mu)}_{\text{root factors in the left half s-plane}} \; \underbrace{\prod^{m} q_\mu \, (a_\mu z^2 = b^*_\mu z + c_\mu)}_{\text{root factors in the right half s-plane}} \tag{VIII.16}$$

related to (VIII.17)

The parameters a_μ, b_μ and c_μ can be calculated from those of equation (VIII.15) in the following manner:

$$\begin{aligned} A_\mu &= (a_\mu^2 + b_\mu^2) \\ B_\mu &= (2\, a_\mu c_\mu - b_\mu^2) \\ C_\mu &= c_\mu^2 \end{aligned} \quad \Longrightarrow \quad \begin{aligned} c_\mu &= +\sqrt{C_\mu} \\ a_\mu &= 1 - c_\mu \\ b_\mu &= +\sqrt{A_\mu - a_\mu^2} \end{aligned} \tag{VIII.18}$$

By means of these decomposition formulas, the polynomials R(z) will assume the form

$$R(z) = C \prod^{n} (z + d^*) \prod^{m} (a_\mu z^2 + b^*_\mu z + c_\mu) \tag{VIII.19}$$

with

$$C = \sqrt{C_q} \prod^{n} (1 - \delta_v^2) \prod^{m} q_\mu^2 \tag{VIII.20}$$

2. The two-port parameters in the z-plane

The polynomial transformations may be applied first to the transfer polynomials and then, by proper combinations, to the rational functions K(s) and H(s). Let

n = degree of F(s) and E(s) $n \leq m$ for lowpass filters,

m = degree of P(s)

then

$$\left.\begin{aligned} F(s) &\rightarrow \frac{\overline{F}(z)}{(\sqrt{1-z^2})^n} \\ E(s) &\rightarrow \frac{\overline{E}(z)}{(\sqrt{1-z^2})^n} \\ P(s) &\rightarrow \frac{\overline{P}(z)}{(\sqrt{1-z^2})^m} \end{aligned}\right\} \qquad \begin{aligned} K(s) &\rightarrow \overline{K}(z) = \frac{\overline{F}(z)}{(\sqrt{1-z^2})^{n-m}\,\overline{P}(z)} \\ H(s) &\rightarrow \overline{H}(z) = \frac{\overline{E}(z)}{(\sqrt{1-z^2})^{n-m}\,\overline{P}(z)} \end{aligned} \qquad \text{(VIII.21)}$$

From the relation

$$H(s)\,H(-s) = 1 + K(s)\,K(-s) \rightarrow \frac{\overline{E}(z)\,\overline{E}(-z)}{(1-z^2)^{n-m}\,\overline{P}(z)\,\overline{P}(-z)} = 1 + \frac{\overline{F}(z)\,\overline{F}(-z)}{(1-z^2)^{n-m}\,\overline{P}(z)\overline{P}(-z)}$$

one may derive the compatibility equation for the z-plane transfer polynomials

$$\overline{E}(z)\,\overline{E}(-z) = \overline{F}(z)\,\overline{F}(-z) + (1-z^2)^{n-m}\,\overline{P}(z)\,\overline{P}(-z) \qquad \text{(VIII.22)}$$

The right hand side of this equation is an even polynomial similar to $Q(z^2)$ in equations (VIII.6)and (VIII.9). Its decomposition yields the polynomial $\overline{E}(z)$ in the form of equation (VIII.19).

In lowpass filters, P(s) is always even. Therefore, the separation of the even and odd parts of $\overline{K}(z)$ and $\overline{H}(z)$ is carried out by separating $\overline{F}(z)$ and $\overline{E}(z)$ in their even and odd parts, respectively. From these, the two-port parameters can be formed (see the table in Fig. VIII.2).

Example. Transpose to the z-plane the characteristic function K(s)

$$K(s) = C\,\frac{(s^2+0.5)}{(s^2+0.5)}$$

where C should be determined such that A = 3.0 dB at s = 0.

	s-plane Functions	z-plane Functions	n = even	n = odd
	$K_e(s)$	$\mathcal{K}_e(z)$	$\dfrac{\mathcal{F}_e}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$	$\dfrac{\sqrt{1-z^2}\;\mathcal{F}_e}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$
	$K_o(s)$	$\mathcal{K}_o(z)$	$\dfrac{\sqrt{1-z^2}\;\mathcal{F}_o(z)}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$	$\dfrac{\mathcal{F}_o}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$
	$H_e(s)$	$\mathcal{H}_e(z)$	$\dfrac{\mathcal{E}_e}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$	$\dfrac{\sqrt{1-z^2}\;\mathcal{E}_e}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$
	$H_o(s)$	$\mathcal{H}_o(z)$	$\dfrac{\sqrt{1-z^2}\;\mathcal{E}_o(z)}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$	$\dfrac{\mathcal{E}_o}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$
a_{11}	$H_e - K_e$	$\mathcal{H}_e - \mathcal{K}_e$	$\dfrac{\mathcal{E}_e - \mathcal{F}_e}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$	$\dfrac{}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$
a_{12}	$H_o - K_o$	$\mathcal{H}_o - \mathcal{K}_o$	$\dfrac{\sqrt{1-z^2}\,[\mathcal{E}_o - \mathcal{F}_o]}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$	$\dfrac{\mathcal{E}_o - \mathcal{F}_o}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$
a_{21}	$H_o + K_o$	$\mathcal{H}_o + \mathcal{K}_o$	$\dfrac{\sqrt{1-z^2}\,[\mathcal{E}_o + \mathcal{F}_o]}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$	$\dfrac{\mathcal{E}_o + \mathcal{F}_o}{\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$
a_{22}	$H_e + K_e$	$\mathcal{H}_e + \mathcal{K}_e$	$\dfrac{[\mathcal{E}_e + \mathcal{F}_e]}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$	$\dfrac{\sqrt{1-z^2}\,[\mathcal{E}_e + \mathcal{F}_e]}{(\sqrt{1-z^2})^{n-m}\,\mathcal{P}(z)}$
z_{11}	$= \dfrac{a_{11}}{a_{21}}$		$\dfrac{[\mathcal{E}_e - \mathcal{F}_e]}{\sqrt{1-z^2}\,[\mathcal{E}_o + \mathcal{F}_o]}$	$\dfrac{\sqrt{1-z^2}\,[\mathcal{E}_e - \mathcal{F}_e]}{[\mathcal{E}_o + \mathcal{F}_o]}$
z_{22}	$= \dfrac{a_{22}}{a_{21}}$		$\dfrac{[\mathcal{E}_e + \mathcal{F}_e]}{\sqrt{1-z^2}\,[\mathcal{E}_o + \mathcal{F}_o]}$	$\dfrac{\sqrt{1-z^2}\,[\mathcal{E}_e + \mathcal{F}_e]}{[\mathcal{E}_o + \mathcal{F}_o]}$
y_{11}	$= \dfrac{a_{22}}{a_{12}}$		$\dfrac{[\mathcal{E}_e + \mathcal{F}_e]}{\sqrt{1-z^2}\,[\mathcal{E}_o - \mathcal{F}_o]}$	$\dfrac{\sqrt{1-z^2}\,[\mathcal{E}_e + \mathcal{F}_e]}{[\mathcal{E}_o - \mathcal{F}_o]}$
y_{22}	$= \dfrac{a_{11}}{a_{12}}$		$\dfrac{[\mathcal{E}_e - \mathcal{F}_e]}{\sqrt{1-z^2}\,[\mathcal{E}_o - \mathcal{F}_o]}$	$\dfrac{\sqrt{1-z^2}\,[\mathcal{E}_e - \mathcal{F}_e]}{[\mathcal{E}_o - \mathcal{F}_o]}$

$\mathcal{K} \equiv \overline{K} \qquad \mathcal{F} \equiv \overline{F} \qquad \mathcal{E} \equiv \overline{E} \qquad \mathcal{P} \equiv \overline{P}$

Figure VIII.2 Table of z-plane Transforms

(a) Transposition of the reflection zero and the attenuation pole.

$$\left.\begin{array}{lll} s^2 = -0.5 & \rightarrow & z = \pm j \\ s^2 = -2.0 & \rightarrow & z = \pm\sqrt{2} \\ s = 0 & \rightarrow & z = 0 \end{array}\right\} \qquad \overline{K}(z) = C\,\frac{(z^2+1)}{(z^2-1)}$$

To make $A(0) = 3.\mathrm{dB} \rightarrow C = -2$

(b) Transfer polynomials

$$\overline{F}(z) = (z^2 + 1); \; \overline{P}(z) = -0.5\,(z^2 - 2)$$

$$\overline{F}(z)\,\overline{F}(-z) + \overline{P}(z)\,\overline{P}(-z) = (z^4 + 2\,z^2 + 1) + 0.25\,(z^4 - 4\,z^2 + 4)$$

$$= 1.25\,z^4 + z^2 + 2$$

$$\overline{E}(z)\,\overline{E}(-z) = 4.25\,[\frac{1.25}{4.25}\,z^4 + \frac{1}{4.25}\,z^2 + \frac{2}{4.25}] = q_v^2[A_v\,z^4 + B_v\,z^2 + C_v$$

$q_v = 2.061$; $A_v = 0.2941$; $B_v = 0.2352$; $C_v = 0.4704$

(VIII.18): $c_v = 0.6858$; $a_v = 0.3141$; $b_v = 0.4420$

$$\overline{E}(z) = 2.061\,[0.3141\,z^2 + 0.4420\,z\,\sqrt{1 - z^2} + 0.6858]$$

$$\overline{H}(z) = \frac{\overline{E}(z)}{\overline{P}(z)} = \frac{-13.12[z^2 + 1.4072\,z\sqrt{1 - z^2} + 2.183]}{(z^2 - 2)}$$

Of the two-port parameters, the expression for z_{11} becomes

$$z_{11} = \frac{\overline{E}_e - \overline{F}_e}{\sqrt{1-z^2}\,\overline{E}_o} = \frac{15.12z^2 + 30.647}{18.46\,z\,\sqrt{1-z^2}}$$

3. Approximation in the z-plane

In order to exploit fully the advantages offered by the z-plane, it is us to carry out the entire design in the z-plane, including the approximation. T this end, the specifications for the stopband are transposed to the real axis of the z-plane. The task of finding suitable locations for the attenuation po is simplified by the close relation to Rumpelt's template method: The absciss γ of this template is the logarithm of the frequencies on the real axis.

After a suitable set of these attenuation poles has been found, it is also considerably easier to determine a polynomial $\overline{F}(z)$ which yields an equal-ripple variation of the transducer loss in the passband, i.e. over the entire j-axis in the z-plane. To this end, one has only to transpose the pertinent formulas of subsection V.2 to the z-plane.

Fig. V.9: $$q_i(s) = \frac{m_i\, s}{\sqrt{1+s^2}} \rightarrow q_i(z) = m_i\, z \qquad \text{(VIII.23)}$$

(V.46):
$$\frac{q(z)+1}{q(z)-1} = \frac{m_1 z+1}{m_1 z-1}\,\frac{m_2 z+1}{m_2 z-1}\,\frac{m_3 z+1}{m_3 z-1} \cdots\cdots$$

For instance, the composition for two attenuation poles yields

$$\frac{q+1}{q-1} = \frac{(m_1 z+1)(m_2 z+1)}{(m_1 z-1)(m_2 z-1)} = \frac{m_1 m_2 z^2 + (m_1+m_2)z+1}{m_1 m_2 z^2 - (m_1+m_2)z+1}$$

$$q(z) = \frac{z^2 + (m_1 m_2)^{-1}}{(m_1^{-1}+m_2^{-1})\, z}$$

(V.48):
$$\overline{K}_o(z) = \frac{[z^2+(m_1 m_2)^{-1}]^2 + [m_1^{-1}+m_2^{-1}]^2\, z^2}{[z^2+(m_1 m_2)^{-1}]^2 - [m_1^{-1}+m_2^{-1}]^2\, z^2}$$

(V.49):
$$\overline{K(z)} = C\,\frac{z^4 + [4(m_1 m_2)^{-1} + m_1^{-2} + m_2^{-2}]z^2 + (m_1 m_2)^{-2}}{(z^2 - m_1^{-2})(z^2 - m_2^{-2})}$$

4. Realization procedure in the z-plane

After having arrived at a suitable set of two-port parameters, one could return to the s-plane to carry out the realization procedure. However, to exploit fully the advantages of the z-plane the realization should also be carried out in this plane.

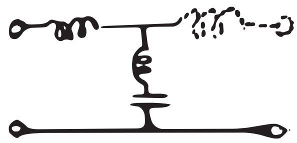
The decomposition of a ladder network in the s-plane is conventionally carried out by the eight subroutines of Fig. VII.9. In all these subroutines, the element values are calculated by the evaluation of even rational functions at a particular pole. For instance, a series inductor is calculated by

$$\ell_i = \lim_{s\to\infty} \frac{1}{s}\, z(s) \text{ in case of a full removal of an attenuation pole}$$

$$\ell_i = \lim_{s^2=-\omega_\infty^2} \frac{1}{s}\, z(s) \text{ in case of a partial removal of an attenuation pole}$$

In both cases, the expression on the right side is an even function. The same is true for all subsequent steps. For this reason, one can not expect complications due to factor $\sqrt{1-z^2}$ which appears in all two-port parameters. For instance, in the numerical example above one proceeds in the following manner to calculate the circuit elements:

$$z_{11} = \frac{\bar{E}_e - \bar{F}_e}{\sqrt{1-z^2}\;\bar{E}_o} = \frac{15.12\, z^2 + 30.647}{18.46\, z\sqrt{1-z^2}} \rightarrow$$



$$\ell_1 = \frac{\sqrt{1-z^2}}{z}\, z_{11}\Bigg|_{z^2=2} = \frac{15.12\, z^2 + 30.647}{18.46\, z^2}\Bigg|_{z^2=2} = 1.649$$

$$z_2 = z_{11} - \frac{\ell_1\, z}{\sqrt{1-z^2}} = \frac{-0.8302\, z^2 + 1.66}{z\sqrt{1-z^2}}$$

$$z_2 = \frac{\dfrac{z^2}{\sqrt{1-z^2}}\,\ell_2 + c_2^{-1}}{\dfrac{z}{\sqrt{1-z^2}}} = \frac{(\ell_2 + c_2^{-1})\, z^2 + c_2^{-1}}{z\sqrt{1-z^2}}$$

$c_2 = 0.6024$, $\ell_2 = 0.830$

ℓ_3 can be calculated from the condition for perfect coupling between the three coils.

5. General z-plane transformations

The preceding subsections describe in rather sketchy form the main concept of z-plane transformations. As discussed, the transformation applies only to lowpass filters. A more general transformation is of the form

$$z = \sqrt{\frac{a^2 + s^2}{1 + s^2}}\;;\quad 0 \le |a| < 1 \tag{VIII.24}$$

which maps the range $j\,|a| \leq s = j\omega \leq j$ and its counterpart on the negative j-axis onto the entire imaginary axis of the z-plane. Obviously, for $|a| > 0$, this is the passband range of a bandpass; for $a = 0$, the transformation formula degenerates to equation (VIII.1), the lowpass case. In great detail, the entire design procedure for this transformation has been described by Orchard and Temes in the December issue of the 1968 Transactions on Circuit Theory. According to Temes, the z-plane transformation offers an advantage of approximately 3 : 1 in precision requirements over conventional s-plane design methods.

An alternative to the z-plane is an s-plane design method which has been suggested by Skwirzinski [SK-1] . According to this method, the various polynomials are not defined and stored by their coefficients but by their roots and a constant multiplier. The precision requirements are also drastically reduced, supposedly almost as much as with z-plane methods.

If one wants to invest a considerable effort in writing a versatile computer program for the synthesis of transmission networks, one should design such a program for degrees up to 40. From experience, the program will then be adequate to cover even the most complicated transmission networks in multiplex telephone systems. Regarding precision, the choices are between conventional s-plane design methods for which a precision of up to 35 significant digits is sufficient in most cases. With the other two methods, regular double precision with 16 significant digits is adequate.

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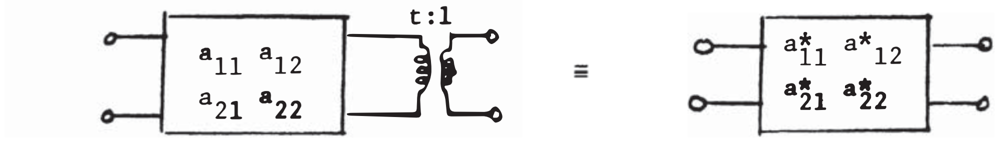
Appendix A: Norton Transformation ([NO-1])

1. **The general concept**



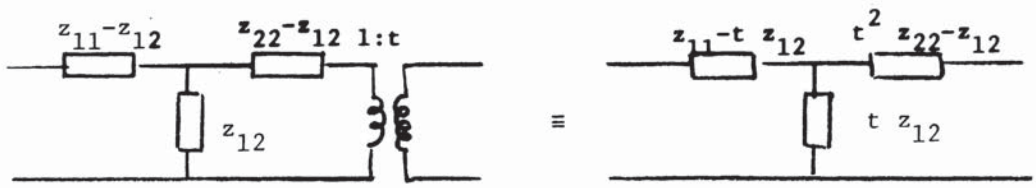
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} t & o \\ o & t^{-1} \end{pmatrix} = \begin{pmatrix} a^*_{11} & a^*_{12} \\ a^*_{21} & a^*_{22} \end{pmatrix} = \begin{pmatrix} a_{11}\, t & a_{12}/t \\ a_{21}\, t & a_{22}/t \end{pmatrix}$$

$$z_{11} = \frac{a_{11}}{a_{21}} \quad ; \quad y_{11} = \frac{a_{22}}{a_{12}} \qquad\qquad z^*_{11} = \frac{a^*_{11}}{a^*_{21}} = z_{11} \quad ; \quad y^*_{11} = \frac{a^*_{22}}{a^*_{12}} = y_{11}$$

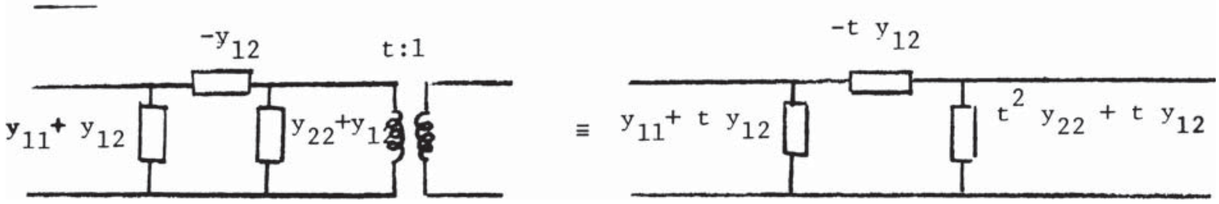
$$z_{12} = \frac{1}{a_{21}} \quad ; \quad -y_{12} = \frac{1}{a_{12}} \qquad\qquad z^*_{12} = \frac{1}{a^*_{21}} = \frac{z_{12}}{t} \quad ; \quad -y^*_{12} = \frac{1}{a^*_{12}} = -y_{12}\, t$$

$$z_{22} = \frac{a_{22}}{a_{21}} \quad ; \quad y_{22} = \frac{a_{11}}{a_{12}} \qquad\qquad z^*_{22} = \frac{a^*_{22}}{a^*_{21}} = \frac{z_{22}}{t^2} \quad ; \quad y^*_{22} = \frac{a^*_{11}}{a^*_{12}} = y_{22}\, t^2$$

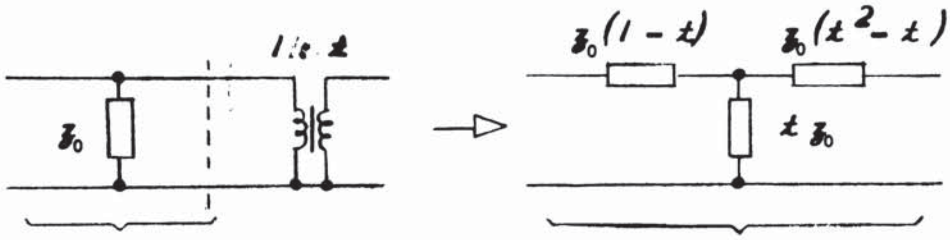
Therefore,



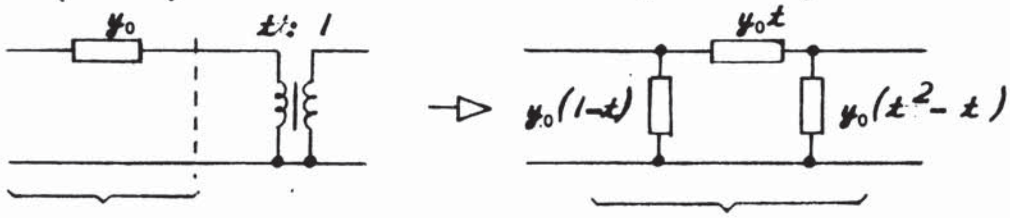
Note: reversal of transformer!



2. The Norton transformation proper.



$$(z) = \begin{pmatrix} z_0 & z_0 \\ z_0 & z_0 \end{pmatrix} \qquad (z^*) = \begin{pmatrix} z_0 & z_0 t \\ z_0 t & z_0 t^2 \end{pmatrix}$$



$$(y) = \begin{pmatrix} y_0 & -y_0 \\ -y_0 & y_0 \end{pmatrix} \qquad (y^*) = \begin{pmatrix} y_0 & -y_0 t \\ -y_0 t & y_0 t^2 \end{pmatrix}$$

Appendix B. Input Data for the Computer Program SYNTH.

1. Input data for SYNTH1

The part SYNTH1 of the overall computer program carries out the calculation of all transfer polynomials and the evaluation of up to 4 transfer performances. It can either start from the poles and zeroes of K(s) or those of H(s). The selection of either mode and also other logical decisions in the program are made from key-words rather than code numbers. This simplifies the application.

In the K(s) mode, the essential input data are:

(a) an arbitrary set of reflection zeroes from which the polynomial F(s) is formed.

(b) an arbitrary set of attenuation poles from which the polynomial P(s) is formed.

The constant C_k is derived from the specified loss A_o (dB) at a specified frequency f_o (kHz). It is necessary that the degree $P(s) \leq$ degr $F(s) \leq 19$.

In the H(s) mode, the essential input data are:

(a) an arbitrary set of natural modes from which the polynomial E(s) is formed.

(b) an arbitrary set of attenuation poles from which the polynomial P(s) is formed.

The constant C_h is derived from the specified loss A_c (dB) at the absolute minimum of H(s). It is advisable to assign a small but finite loss A_o (dB), for instance 0.001 dB. It is necessary that degr $P(s) \leq$ degr $E(s) \leq 19$ and, furthermore, that degree $E(s)\,E(-s)\,P\,(s) \leq 38$, otherwise, the root program will not be able to find the roots of the derivative of H(s).

In either mode, two of the three transfer polynomials are specified. The third is then calculated from the compatibility equation

$$E(s)\,E(-s) = F(s)\,F(-s) + P(s)\,P(-s)$$

With all transfer polynomials well established the following transfer performances are carried out:

the transducer loss - keyword " LOSS "
the phase response - keyword " PHAS "
the delay response - keyword " DELA "
the step response - keyword " STEP "
the impulse response- keyword " IMPU "

At the option of the user, one to four evaluations can be carried out in one batch. In these evaluations the frequency or time scale can be linear (keyword "LINE") or logarithmic (keyword "LOGA")

<u>Data format for SYNTH1</u>. (See Fig. B.1)

		K(s)-mode	H(s)-mode
Crd. 1	col.02-40	Arbitrary title	Arbitrary Title
	col.70	keyletter " K "	keyletter " H "
Crd. 2	col.10-11	degree F(s)	degree E(s)
	col.30-31	no.of refl.zeros at origin	no.of complex root pairs
	col.50-51	no.of pairs of refl zeroes	no.of E(s) roots on real axis
	col.70-71	no.of refl.zeroes real	ignored.
Crd. 3	col.30-31	no.of atten.poles at the origin	
	col.50-51	no.of atten.pole pairs on either axis	
	col.70-71	no.of atten.pol quadruplets	
Crd. 4	col.10-17	reference frequency (kHz)	
	col.30-37	A_o(dB) as discussed above	
	col.50-57	f_o(kHz) as discussed above	ignored
	col.70-77	the real part σ (kHz) of a frequency which is used in the evaluation of the transducer loss. Default σ=0.0	

Crd. 5	col.10-13,30-33,50-53,70-73	keywords LOSS,PHASE,... etc	
Crd. 6	col.10-17,30-37,50-57,70-77	minim.	of the evaluation range
Crd. 7	col.10-17,30-37,50-57,70-77	maxim.	of the evaluation range
Crd. 8	col.10-13,30-33,50-53,70-73	scale.	of the evaluation range
Crd. 9	col.10-17,30-37,50-57,70-77	increm.	of the evaluation range
Crd.10	col. 2-40	arbitrary. suggested : "PLOT MARGINS"	
Crd.11	col.10-17,30-37,50-57,70-77	left margin	
Crd.12	col.10-17,30-37,50-57,70-77	right margin	plot range
Crd.13	col.10-17,30-37,50-57,70-77	subdivis.of	
Crd.14	col. 2-40	arbitrary. Suggested "REFL:ZEROES" or"NATURAL MODES"	

		K(s)-mode	H(s)-mode
Crd.15a	col.10-22	real part of refl.zero	real part of natural mode
	col.30-42	imagin.part of refl.zero	imag.part of natural mode

As many cards as needed for all refle.zeroes or natural modes. Do not include those at the origin. One member of a pair only:

Crd.16	col. 2-40	arbitrary. Suggested : "ATTENUATION POLES"
Crd.17a	col.10-22	real part of attenuation poles
	col.30-42	imaginary part of attenuation poles

```
TYPICAL DATA FOR SYNTH1   K(S) DESIGN                                   PAGE   OF   PAGES

EXAMPLE NO.1 CAUER PAR. LP C 07-10 60                                           K(S) DESIGN
DEGREE.  7  REFL. ZEROES      1 AT THE ORIGIN       3 PAIRS                     0 REAL
            ATTEN. POLES      0 AT THE ORIGIN       3 PAIRS                     0 QUADS
REF. FRQ   10.0   KHZ         42.66  DB  AT        11.547  KHZ
EVAL.   LOSS RESPONSE         LOSS RESPONSE         PHASE RESPONSE              STEP RESP.
FROM        0.0   KHZ          10.0   KHZ             0.0  KHZ                   0.001  MS
TO         10.0   KHZ          40.0   KHZ            20.0  KHZ                   1.0    MS
OVER A  LINEAR SCALE          LOGARITHM. SCALE      LINEAR SCALE                LOGAR. SCALE
WITH        0.2   KHZ INCR     20.0   FRQ/DEC.        0.4  KHZ INCR.             20.0   FD
PLOT MARGINS
LEFT        0.0   DB            0.0   DB              0.0  DGR                  -0.2    V
RIGHT       1.0   DB.         100.0   DB            810.0  DGR                   1.8    V
SUBDIV.     0.2   DB           10.0   DB             90.0  DGR                   0.2    V
REFL. ZER.  RE(KHZ)               IM(KHZ)
      A1=  0.0               B1=  5.55795
      A2=  0.0               B2=  8.72436
      A3=  0.0               B3=  9.87878
ATTEN. PLS.  RE(KHZ)              IM(KHZ)
      X1=  0.0               Y1=  11.688694836
      X2=  0.0               Y2=  13.235361916
      X3=  0.0               Y3=  20.775633265
```

```
TYPICAL DATA FOR SYNTH1   H(S) DESIGN                                   PAGE   OF   PAGES

PHASE LINEAR LP (ULBRICH-PILOTY)                                                H(S) DESIGN
DEGREE    5  NATURAL MODES    2 PAIRS               1 REAL
             ATTEN. POLES     0 AT THE ORIGIN       0 PAIRS                     0 QUADR
REF. FRQ.  10.0   KHZ     A0= 0.00[illegible]  DB AT MIN.
EVAL    LOSS RESPONSE         PHASE RESPONSE        DELAY RESPONSE
FROM        0.0   KHZ         0.0      KHZ          0.0     KHZ
TO        100.0   KHZ         10.0     KHZ          10.0    KHZ
OVER A  LINEAR RANGE          LINEAR RANGE          LINEAR RANGE
WITH        2.0   KHZ INCR    0.2      KHZ INCR.    0.2     KHZ INCR.
PLOT MARGINS
LEFT        0.0   DB          0.0      DEGR         0.0     MSEC
RIGHT     100.0   DB          540.0    DEGR         0.1     MSEC
SUBDIV.    20.0   DB          90.0     DEGR         0.01    MSEC
NAT. MODES  RE(KHZ)               IM(KHZ)
      A1=-4.54219            B1=0.0
      A2=-4.35686            B2=5.229[illegible]2
      A3=-3.43478            B3=10.11568
ALL ATTEN. POLES AT INFINITY
```

Figure B.1 Typical Data for SYNTH 1

(The data in the unshaded areas are essential. Data in shaded areas is arbitrary and can be used for convenience as one pleases. Most of it will be duplicated in the same form on the printer.)

139

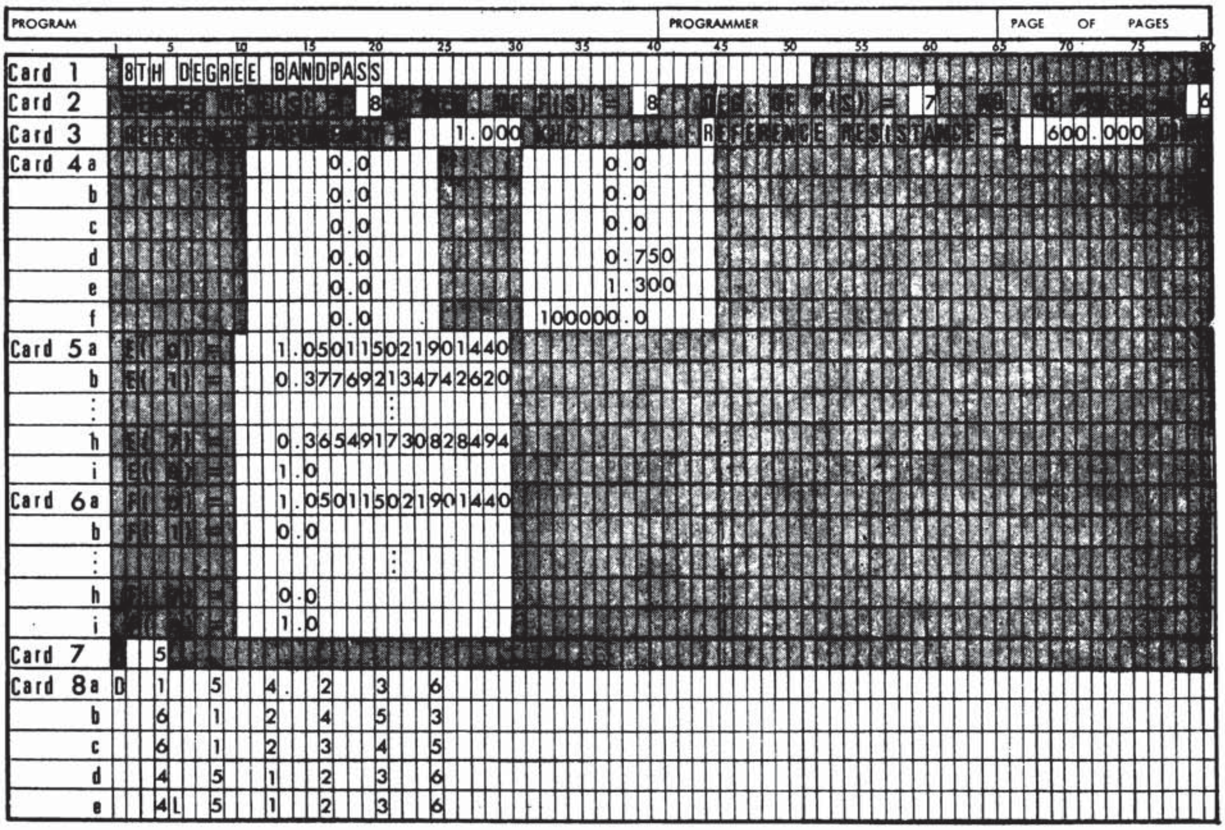
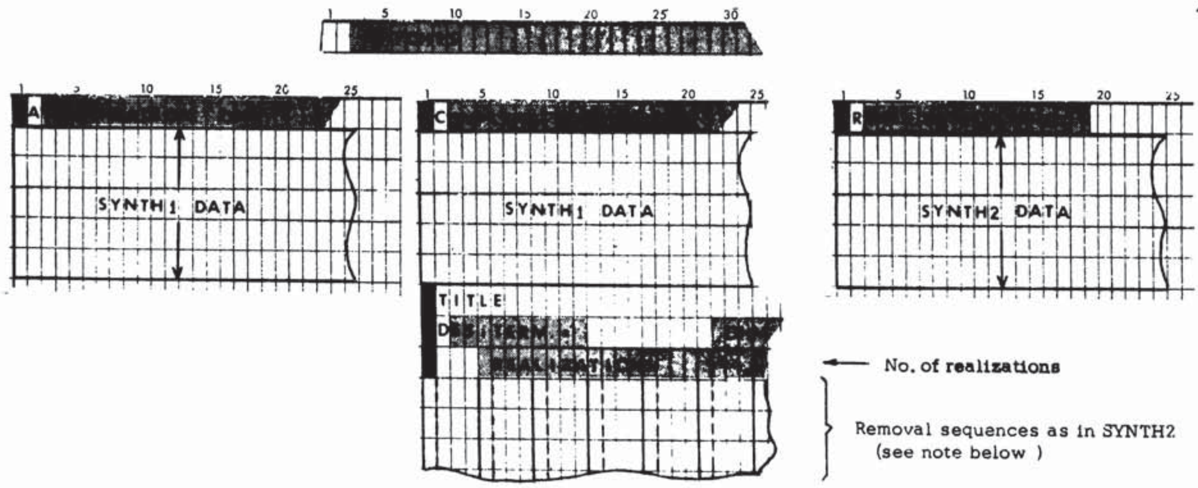


Figure B.2 Typical Data Format for SYNTH 2

Data cards for SYNTH



Note: In the transition from SYNTH1 to SYNTH2 , the list of stored attenuation poles is augmented by those at zero and infinity. Those at zero will precede and those at infinity will follow the the list of previously stored poles.

Figure B. 3 Typical Data Format for SYNTH

As many cards as needed. Do not include those at the origin and at infinity. One member of a pair or a quadruplet only.

2. Input data for SYNTH2

If used independently, the program synthesis needs the coefficients of F(s) E(s) and also the degree of P(s). It also needs the locations of all attenuation poles, including those at zero and infinity.

For the actual removal of all attenuation poles in the realization procedure, one must also specify one or several sequences by which these poles are to be removed. Each removal sequence yields uniquely one ladder circuit. (Except for subsequent network transformations.)

Data format for SYNTH2 (See Fig. B.2)

Crd. 1	col. 2-51	Title
Crd. 2	col.19-20	degr E(s) col.39-40 degr F(s) col.59-60 degr P(s)
	col.79-80	no.of atten.poles.
Crd. 3	col.23-30	reference freq. (kHz) col.67-75 refer.resistance.
Crd 4a	col.11-24	real part of all attenuation poles
	col.31-44	imag.part

As many cards as necessary. List all poles, including those at the origin. Include only one member of a pair or a quadruplet. Attenuation poles at infinity are indicated by a real part 0.0 and an imaginary part $\geq$ 100 000. The number of poles must agree with the number specified on card 2.

Crd. 5a	col.10-29	coefficients of E(s) in ascending order
b		
c		
Crd. 6a	col.10-29	coefficients of F(s) in ascending order
b		
Crd. 7	col.2-4	no. desired realizations
Crd. 8a	col. 1	blank or letter D. In this case the circuit will also be denormalized.

col.2-4,6-8,10-12,14-16,etc. removal sequence of atten.poles. The numbers in these fields corresponds to the number in which the pertinent attenuation poles has been fed on a data card. There must be as many sequence cards as indicated on card 7.

In the removal procedure of attenuation poles, SYNTH2 takes all necessary precautions which are discussed in section VII. For instance, one may by intention or by mistake remove first all the attenuation poles at zero or at infinity or both. Then, if the removal of an attenuation pole at a finite frequency requires first the partial removal of a pole at these frequencies the removal will be carried out by a Brune section. The user has also the option to force the removal of such a section by making the related sequence number negative.

3. Input data for SYNTH

In the version which is available from N. C. State and also as a service from CDC Cybernet, the two major parts SYNTH1 and SYNTH2 have been combined into a comprehensive computer program SYNTH. This program is set up to carry out consecutively several batches of synthesis calculations. Fig. B.3 shows the data cards for these batches.

Crd. 1 col. 1 - 2 Integer indicating the total number of batches.

Crd. 2 col. 2 Letter "A","R" or "C". This card should precede each batch of data cards.

As rather obvious from Fig. B.3. the letters "A" and "R" carry out a synthesis procedure as described in detail above. The letter "C" carries out a complete synthesis. After the completion of SYNTH1, there is a transition program which leads directly into the realization part proper of SYNTH2. In this transition part the specified list of attenuation poles is complemented by those at zero and at infinity. Those at zero will precede, those at infinity follow the attenuation poles specified in SYNTH1. All coefficients of the transfer polynomials are carried over. Therefore, it is only necessary to add the following cards after the SYNTH1 data:

Crd 3 col. 2-40 Title

Crd. 4 col. 2 letter " D " or " R " (not necessary in the expanded CDC-Cybernet version)

col.14-21 reference resistance for the denormalization.

Crd. 5 and all following cards control the realization and are identical to cards 7, 8a, 8b,.... of SYNTH2.

In all coding forms of Fig. B.1,2, and 3, the essential data goes into the unshaded part of the forms. In the shaded portions, the user may insert any convenient information. This aids the identification of data cards in the deck. Most of this data in shaded areas will also be duplicated on the printer but otherwise ignored. A typical print-out of a resulting bandpass circuit is shown in Fig. B.4

REALIZATION SEQUENCE (1)

REMOVAL SEQUENCE

[illegible]

NORMALIZED CIRCUIT — DUAL CIRCUIT

R(1) =	0.1000000000 01	= G(1)
L(2) =	0.17983370D 00	= C(2)
C(3) =	[illegible] D 01	= L(3)
C(4) =	[illegible] D 01	= L(4)
L(4) =	[illegible] D 00	= C(4)
W(4) =	0.12999992E 01	= W(4)
C(5) =	[illegible] 01	= L(5)
C(6) =	0.2008[illegible]D 01	= L(6)
L(6) =	0.8852[illegible]D 00	= C(6)
W(6) =	0.75000000E 00	= W(6)
C(7) =	0.357[illegible] 01	= L(7)
L(8) =	0.6392[illegible]D-01	= C(8)
C(9) =	0.1473[illegible] 02	= L(9)
R(10) =	0.3554[illegible] 00	= G(10)

(a)

REALIZATION SEQUENCE (1)

REMOVAL SEQUENCE

C 1 5 4 2 3 [illegible]

DENORMALIZED ELEMENT VALUES

R(1) =	[illegible] OHMS
L(2) =	[illegible] MH
C(3) =	[illegible] NF
C(4) =	[illegible] NF
L(4) =	[illegible] MH
F(4) =	[illegible] KHZ
C(5) =	[illegible] NF
C(6) =	[illegible] NF
L(6) =	[illegible] MH
F(6) =	[illegible] KHZ
C(7) =	[illegible] NF
L(8) =	[illegible] MH
C(9) =	[illegible] NF
R(10) =	[illegible] OHMS

(b)

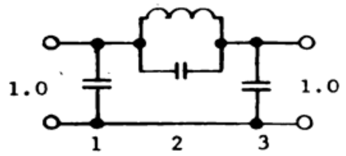
Figure B.4 The Typical Realization of a Bandpass Circuit

Design Tables

for

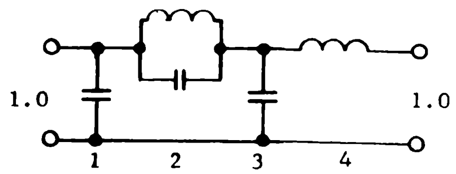
Inverted Chebyshev Filters

(These Tables were calculated and compiled by F.W. Shepherd)



N=3

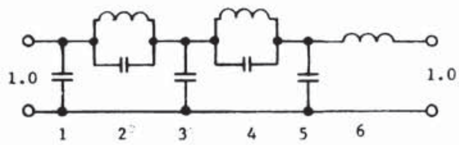
AMIN	C1	C2	L2	F2	C3
10.0	.6439	.5823	1.2879	1.154700	.6439
15.0	.8935	.4196	1.7870		.8935
20.0	1.1717	.3200	2.3434		1.1717
25.0	1.4917	.2513	2.9835		1.4917
30.0	1.8664	.2009	3.7328		1.8664
35.0	2.3098	.1623	4.6196		2.3098
40.0	2.8384	.1321	5.6769		2.8384
45.0	3.4719	.1080	6.9438		3.4719
50.0	4.2336	.0885	8.4672		4.2336
55.0	5.1516	.0727	10.3032		5.1516
60.0	6.2599	.0599	12.5198		6.2599
65.0	7.5993	.0493	15.1987		7.5993



N=4

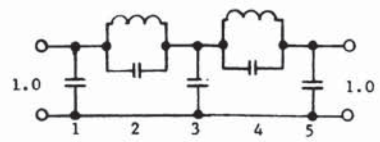
AMIN	C1	C2	L2	F2	C3	L4
15.0	.3248	.6892	1.2018	1.098684	1.6916	.8146
20.0	.5123	.5266	1.5730		1.9894	.9287
25.0	.7036	.4194	1.9751		2.3305	1.0590
30.0	.9056	.3427	2.4171		2.7215	1.2100
35.0	1.1242	.2848	2.9086		3.1699	1.3856
40.0	1.3648	.2394	3.4600		3.6848	1.5896
45.0	1.6328	.2029	4.0828		4.2765	1.8265
50.0	1.9342	.1729	4.7901		4.9573	2.1013
55.0	2.2753	.1480	5.5967		5.7410	2.4196
60.0	2.6634	.1270	6.5192		6.6439	2.7881
65.0	3.1066	.1093	7.5769		7.6847	3.2144
70.0	3.6142	.0942	8.7918		8.8851	3.7074

N=6



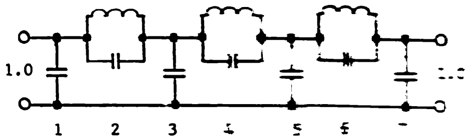
AMIN	C1	C2	L2	F2	C3	C4	L4	F4	C5	L6
25.0	.1530	.4621	1.0042	1.467890	1.6006	.8255	1.1243	1.037955	.9045	.5295
30.0	.2375	.3986	1.1642		1.7855	.6751	1.3748		1.0910	.5751
35.0	.3223	.3474	1.3355		1.9938	.5671	1.6367		1.2817	.6256
40.0	.4087	.3053	1.5200		2.2252	.4853	1.9124		1.4804	.6818
45.0	.4978	.2699	1.7193		2.4804	.4209	2.2048		1.6900	.7442
50.0	.5908	.2398	1.9351		2.7607	.3688	2.5165		1.9134	.8133
55.0	.6887	.2139	2.1691		3.0680	.3255	2.8508		2.1530	.8899
60.0	.7926	.1914	2.4235		3.4048	.2890	3.2109		2.4116	.9746
65.0	.9035	.1718	2.7006		3.7738	.2578	3.6001		2.6917	1.0683
70.0	1.0225	.1545	3.0027		4.1782	.2307	4.0222		2.9961	1.1718
75.0	1.1507	.1392	3.3327		4.6214	.2071	4.4812		3.3277	1.2860
80.0	1.2894	.1256	3.6934		5.1076	.1863	4.9812		3.6897	1.4121

N=5



AMIN	C1	C2	L2	F2	C3	C4	L4	F4	C5
20.0	.3924	.2586	1.3356	1.701302	1.8141	1.2573	.7193	1.051462	-.1515
25.0	.4798	.2249	1.5360		1.9977	.9263	.9763		.0348
30.0	.5733	.1971	1.7520		2.2279	.7234	1.2502		.2009
35.0	.6744	.1736	1.9895		2.4992	.5865	1.5421		.3579
40.0	.7845	.1533	2.2528		2.8109	.4875	1.8550		.5123
45.0	.9049	.1357	2.5459		3.1645	.4125	2.1926		.6690
50.0	1.0374	.1202	2.8729		3.5630	.3534	2.5591		.8316
55.0	1.1836	.1066	3.2382		4.0107	.3056	2.9594		1.0032
60.0	1.3455	.0947	3.6466		4.5130	.2661	3.3988		1.1868
65.0	1.5253	.0841	4.1035		5.0761	.2329	3.8833		1.3853
70.0	1.7253	.0748	4.6150		5.7071	.2046	4.4191		1.6016
75.0	1.9482	.0665	5.1878		6.4143	.1804	5.0135		1.8386

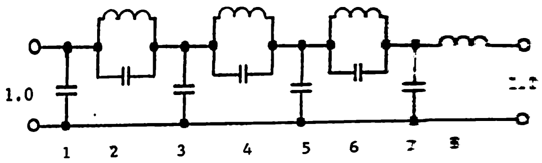
N =7



AMIN	C1	C2	L2	F2	C3	C4	L4	F4	C5	C6	L6	F6	C7
30.0	.2793	.1791	1.0510		1.3413	.7849	1.2108		1.4611	1.0938	.5588		-.2611
35.0	.3235	.1632	1.1532		1.5075	.6586	1.4215		1.5791	.8828	.6923		-.1430
40.0	.3699	.1490	1.2630		1.6826	.5799	1.6388		1.7211	.7331	.8337		-.0381
45.0	.4188	.1362	1.3815		1.8683	.5096	1.8649		1.8838	.6220	.9826		.0584
50.0	.4705	.1247	1.5094		2.0658	.4521	2.1020		2.0656	.5360	1.1403		.1497
55.0	.5254	.1142	1.6477		2.2768	.4040	2.3521		2.2655	.4683	1.3051		.2384
60.0	.5839	.1047	1.7973	2.304765	2.5027	.3631	2.6172	1.025717	2.4843	.4127	1.4809	1.279048	.3257
65.0	.6463	.0960	1.9591		2.7452	.3277	2.8995		2.7226	.3663	1.6683		.4128
70.0	.7131	.0882	2.1343		3.0061	.2969	3.2010		2.9808	.3275	1.8662		.5014
75.0	.7848	.0810	2.3241		3.2872	.2697	3.5239		3.2603	.2943	2.0767		.5924
80.0	.8617	.0744	2.5296		3.5903	.2455	3.8703		3.5630	.2655	2.3017		.6865
85.0	.9445	.0683	2.7522		3.9177	.2240	4.2428		3.8905	.2404	2.5423		.7846

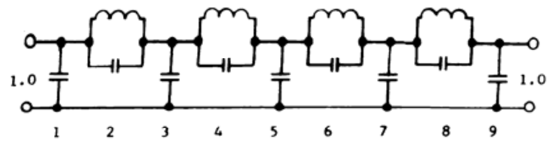
2.304765 1.025717 1.279048

N=8



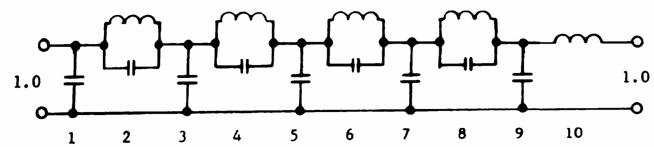
AMIN	C1	C2	L2	F2	C3	C4	L4	F4	C5	C6	L6	F6	C7	L8
35.0	.0968	.3556	.7908		1.1862	.8164	1.1763		1.4580	.6733	1.0085		.6269	.3923
40.0	.1448	.3194	.8806		1.3260	.7064	1.3596		1.6029	.5868	1.1571		.7406	.4170
45.0	.1927	.2883	.9757		1.4718	.6202	1.5484		1.7607	.5174	1.3124		.8551	.4439
50.0	.2409	.2613	1.0763		1.6246	.5506	1.7440		1.9311	.4604	1.4750		.9716	.4729
55.0	.2898	.2377	1.1829		1.7855	.4930	1.9477		2.1139	.4126	1.6456		1.0914	.5044
60.0	.3399	.2170	1.2960	1.885435	1.9554	.4444	2.1609	1.020390	2.3094	.3721	1.8249	1.213455	1.2155	.5384
65.0	.3913	.1986	1.4161		2.1354	.4027	2.3847		2.5183	.3372	2.0138		1.3448	.5753
70.0	.4445	.1822	1.5438		2.3264	.3665	2.6204		2.7412	.3068	2.2132		1.4804	.6150
75.0	.4998	.1674	1.6797		2.5295	.3347	2.8694		2.9791	.2801	2.4242		1.6229	.6579
80.0	.5575	.1541	1.8245		2.7456	.3065	3.1331		3.2329	.2564	2.6477		1.7734	.7042
85.0	.6180	.1421	1.9788		2.9759	.2814	3.4127		3.5043	.2353	2.8850		1.9328	.7542
90.0	.6816	.1312	2.1435		3.2217	.2588	3.7199		3.7936	.2164	3.1372		2.1018	.8080

1.885435 1.020390 1.213455



N=9

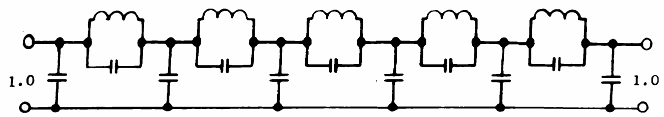
AMIN	C1	C2	L2	F2	C3	C4	L4	F4	C5	C6	L6	F6	C7	C8	L8	F8	C9
40.0	.2164	0.1382	0.8466	2.923804	1.1027	.6038	1.2420	1.154700	1.4313	.9160	1.0586	1.015426	1.0976	.9305	.4440	1.555723	-.2587
45.0	.2431	0.1289	0.9070		1.2148	.5427	1.3818		1.5635	.7973	1.2163		1.1896	.7856	.5259		-.1800
50.0	.2707	0.1202	0.9731		1.3308	.4910	1.5272		1.7055	.7042	1.3770		1.2914	.6753	.6117		-.1092
55.0	.2994	0.1121	1.0434		1.4514	.4467	1.6788		1.8567	.6289	1.5419		1.4020	.5889	.7015		-.0440
60.0	.3294	0.1046	1.1181		1.5774	.4081	1.8373		2.0173	.5664	1.7120		1.5212	.5196	.7951		.0171
65.0	.3607	0.0977	1.1974		1.7093	.3743	2.0034		2.1872	.5136	1.8881		1.6488	.4627	.8928		.0756
70.0	.3935	0.0913	1.2817		1.8478	.3443	2.1777		2.3670	.4682	2.0713		1.7849	.4153	.9947		.1321
75.0	.4279	0.0853	1.3713		1.9936	.3176	2.3609		2.5572	.4286	2.2624		1.9295	.3751	1.1012		.1875
80.0	.4640	0.0798	1.4665		2.1473	.2936	2.5537		2.7583	.3938	2.4624		2.0832	.3407	1.2125		.2424
85.0	.5021	0.0746	1.5678		2.3095	.2720	2.7571		2.9711	.3629	2.6722		2.2462	.3108	1.3290		.2972
90.0	.5422	0.06983	1.6756		2.4811	.2523	2.9717		3.1963	.3352	2.8927		2.4191	.2847	1.4512		.3525
95.0	.5845	0.065	1.7903		2.6627	.2344	3.1984		3.4348	.3103	3.1249		2.6034	.2610	1.5827		.4076



N=10

AMIN	C1	C2	L2	F2	C3	C4	L4	F4	C5	C6	L6	F6	C7	C8	L8	F8	C9	L10
40.0	.0392	0,3156	0,5900	2.317449	.8523	.8138	.9690	1,125998	1.2613	.9270	1.0516	1.012783	1.1178	.6345	.7682	1.43229?	.4053	.2972
45.0	.0704	0,2887	0,6450		.9489	.7223	1.0918		1.3764	.8173	1.1927		1.2270	.5647	.8631		.4815	.3118
50.0	.1013	0,265	0,7026		1.0472	.6474	1.2182		1.4992	.7294	1.3365		1.3413	.5069	.9615		.5569	.3274
55.0	.1321	0,2441	0,7628		1.1480	.5847	1.3487		1.6292	.6570	1.4838		1.4610	.4582	1.0637		.6324	.3440
60.0	.1629	0,2254	0,8261		1.2520	.5316	1.4836		1.7663	.5961	1.6353		1.5863	.4167	1.1697		.7085	.3617
65.0	.1939	0,2087	0,8922		1.3595	.4858	1.6234		1.9105	.5441	1.7916		1.7174	.3808	1.2800		.7859	.3806
70.0	.2254	0,1937	0,9613		1.4712	.4459	1.7684		2.0619	.4990	1.9535		1.8548	.3495	1.3946		.8649	.4007
75.0	.2573	0,1801	1,0339		1.5875	.4109	1.9193		2.2209	.4595	2.1215		1.9988	.3219	1.5141		.9461	.4222
80.0	.2900	0,1677	1,1103		1.7088	.3798	2.0764		2.3878	.4245	2.2963		2.1498	.2974	1.6387		1.0298	.4450
85.0	.3234	0,1565	1,1898		1.8356	.3520	2.2404		2.5630	.3933	2.4785		2.3082	.2755	1.7688		1.1164	.4693
90.0	.3578	0,1461	1,2745		1.9684	.3270	2.4117		2.7471	.3652	2.6688		2.4745	.2558	1.9049		1.2064	.4951
95.0	.3933	0,1366	1,3631		2.1077	.3044	2.5909		2.9405	.3399	2.8678		2.6492	.2380	2.0473		1.3000	.5226

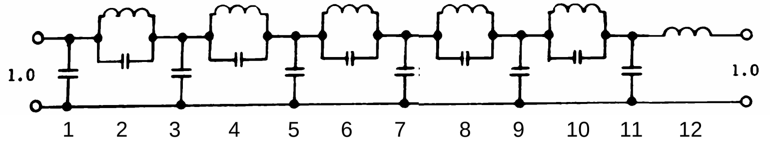
N = 11



AMIN	C1	C2	L2	F2	C3	C4	L4	F4	C5	C6	L6
40.0	.1424	.1272	.6235	3549465	.7767	.6023	0,9481	1,323189	1,1148	1,0047	0,9751
45.0	.1593	.1199	.6616		.8523	.5483	1,0415		1,2185	0,8909	1,0996
50.0	.1766	.1131	.7016		.9292	.5018	1,1380		1,3259	0,7993	1,2257
55.0	.1943	.1067	.7436		1.0078	.4614	1,2377		1,4374	0,7236	1,3539
60.0	.2127	.1007	.7877		1.0886	.4259	1,3410		1,5531	0,6596	1,4851
65.0	.2316	.0951	.8340		1.1718	.3944	1,4481		1,6734	0,6048	1,6198
70.0	.2511	.0899	.8826		1.2578	.3662	1,5592		1,7984	0,5571	1,7585
75.0	.2713	.0849	.9338		1.3469	.3410	1,6747		1,9285	0,5151	1,9016
80.0	.2923	.0803	.9875		1.4394	.3182	1,7949		2,0641	0,4779	2,0498
85.0	.3140	.0760	1.0440		1.5356	.2974	1,9200		2,2055	0,4446	2,2034
90.0	.3366	.0719	1.1034		1.6359	.2785	2,0504		2,3531	0,4146	2,3630
95.0	.3602	.0680	1.1658		1.7404	.2612	2,1864		2,5072	0,3874	2,5289

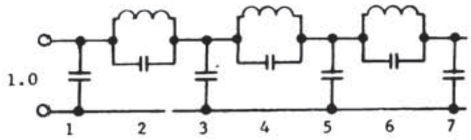
AMIN	F6	C7	C8	L8	F8	C9	C10	L10	F10	C11
40.0	1,010283	1,1029	1,1708	0,7066	1,099345	0,7493	1,1099	0,2633	1,849656	-0,3695
45.0		1,1939	1,0133	0,8165		0,8074	0,9320	0,3136		-0,2985
50.0		1,2931	0,8913	0,9282		0,8714	0,7976	0,3664		-0,2363
55.0		1,3994	0,7940	1,0420		0,9407	0,6932	0,4216		-0,1818
60.0		1,5119	0,7144	1,1581		1,0147	0,6101	0,4790		-0,1302
65.0		1,6305	0,6479	1,2769		1,0934	0,5426	0,5386		-0,0833
70.0		1,7549	0,5915	1,3987		1,1765	0,4868	0,6004		-0,0393
75.0		1,8851	0,5429	1,5240		1,2641	0,4399	0,6643		-0,0250
80.0		2,0213	0,5005	1,6531		1,3560	0,4000	0,7306		0,0427
85.0		2,1636	0,4631	1,7866		1,4525	0,3657	0,7992		0,0818
90.0		2,3123	0,4299	1,9246		1,5537	0,3358	0,8702		0,1200
95.0		2,4677	0,4001	2,0678		1,6596	0,3096	0,9439		0,1577

N = 12



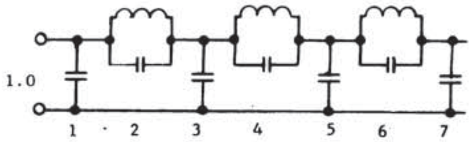
AMIN	C1	C2	L2	F2	C3	C4	L4	F4	C5	C6	L6	F6	C7	C8	L8	F8	C9	C10	L10	F10	C11	C12
40.0	-.0108	.3053	.4311	2,756039	.5781	.8720	0,7144	1,266953	0,9577	1,1375	0,8638	1,008780	1,0411	1,0486	0,8115	1,084005	0,8061	0,6565	0,5478	1,667404	0,2266	0,2301
45.0	.0116	.2820	.4668		.6486	.7791	0,7996		1,0469	1,0059	0,9768		1,1315	0,9311	0,9139		0,8860	0,5883	0,6113		0,2830	0,2392
50.0	.0336	.2612	.5039		.7190	.7023	0,8871		1,1396	0,9007	1,0909		1,2276	0,8357	1,0182		0,9679	0,5314	0,6767		0,3378	0,2487
55.0	.0553	.2427	.5424		.7897	.6378	0,9768		1,2358	0,8143	1,2066		1,3286	0,7566	1,1247		1,0523	0,4832	0,7443		0,3917	0,2589
60.0	.0768	.2260	.5823		.8613	.5828	1,0690		1,3357	0,7419	1,3243		1,4342	0,6898	1,2336		1,1393	0,4418	0,8139		0,4451	0,2695
65.0	.0983	.2110	.6239		.9342	.5354	1,1636		1,4392	0,6802	1,4446		1,5444	0,6326	1,3451		1,2291	0,4060	0,8858		0,4984	0,2808
70.0	.1197	.1973	.6670		1.0087	.4940	1,2611		1,5466	0,6267	1,5678		1,6590	0,5830	1,4596		1,3220	0,3746	0,9600		0,5519	0,2928
75.0	.1412	.1849	.7118		1.0850	.4576	1,3614		1,6580	0,5799	1,6943		1,7782	0,5395	1,5773		1,4182	0,3469	1,0365		0,6060	0,3053
80.0	.1628	.1735	.7584		1.1635	.4253	1,4648		1,7736	0,5385	1,8345		1,9019	0,5010	1,6985		1,5178	0,3223	1,1157		0,6608	0,3186
85.0	.1847	.1631	.8069		1.2444	.3963	1,5720		1,8935	0,5016	1,9587		2,0305	0,4666	1,8235		1,6211	0,3003	1,1975		0,7165	0,3326
90.0	.2069	.1535	.8574		1.3280	.3703	1,6824		2,0180	0,4685	2,0973		2,1640	0,4358	1,9526		1,7284	0,2805	1,2822		0,7735	0,3473
95.0	.2294	.1446	.9098		1.4144	.3467	1,7969		2,1474	0,4385	2,2406		2,3028	0,4079	2,0861		1,8397	0,2625	1,3699		0,8320	0,3629

N = 13

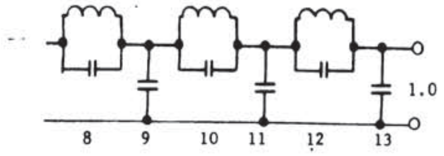


AMIN	C1	C2	L2	F2	C3	C4	L4	F4	C5	C6	L6	F6	C7
40.0	.1009	.1169	.4897	4.178581	.5770	.5886	.7469	1.508016	.8790	1.0365	.8433	1.069500	.9712
45.0	.1126	.1111	.5153		.6318	.5407	.8131		.9590	.9326	.9373		1.0528
50.0	.1245	.1056	.5419		.6870	.4990	.8812		1.0407	.8464	1.0328		1.1394
55.0	.1367	.1005	.5697		.7428	.4623	.9511		1.1243	.7736	1.1300		1.2303
60.0	.1491	.0956	.5986		.7995	.4297	1.0231		1.2101	.7111	1.2294		1.3251
65.0	.1618	.0910	.6287		.8573	.4007	1.0973		1.2982	.6568	1.3310		1.4236
70.0	.1748	.0867	.6601		.9163	.3746	1.1737		1.3888	.6091	1.4351		1.5257
75.0	.1882	.0826	.6929		.9768	.3510	1.2526		1.4822	.5669	1.5419		1.6313
80.0	.2019	.0787	.7270		1.0389	.3296	1.3339		1.5785	.5292	1.6517		1.7406
85.0	.2160	.0750	.7626		1.1028	.3100	1.4180		1.6779	.4954	1.7647		1.8537
90.0	.2306	.0716	.7998		1.1686	.2921	1.5049		1.7806	.4647	1.8811		1.9707
95.0	.2456	.0682	.8385		1.2366	.2757	1.5948		1.8869	.4368	2.0011		2.0919

N = 14

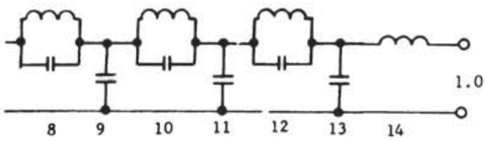


AMIN	C1	C2	L2	F2	C3	C4	L4	F4	C5	C6	L6	F6	C7
40.0	-.0368	.2926	.3340	3.198074	.4066	.9028	.5467	1.423276	.7381	1.2626	.7044	1.060277	.8880
45.0	-.0196	.2723	.3589		.4615	.8108	.6088		.8082	1.1233	.7918		.9629
50.0	-.0029	.2541	.3847		.5154	.7342	.6723		.8800	1.0105	.8802		1.0421
55.0	.0134	.2376	.4114		.5689	.6695	.7373		.9536	.9171	.9698		1.1250
60.0	.0296	.2227	.4389		.6223	.6141	.8038		1.0291	.8384	1.0609		1.2113
65.0	.0456	.2092	.4673		.6759	.5661	.8718		1.1067	.7710	1.1535		1.3008
70.0	.0615	.1968	.4967		.7300	.5243	.9415		1.1864	.7127	1.2480		1.3932
75.0	.0772	.1855	.5270		.7848	.4873	1.0128		1.2683	.6616	1.3444		1.4886
80.0	.0930	.1751	.5583		.8405	.4545	1.0860		1.3526	.6164	1.4429		1.5871
85.0	.1087	.1655	.5907		.8973	.4251	1.1610		1.4394	.5761	1.5437		1.6886
90.0	.1246	.1566	.6242		.9553	.3987	1.2379		1.5288	.5400	1.6471		1.7933
95.0	.1405	.1483	.6588		1.0148	.3748	1.3171		1.6209	.5073	1.7532		1.9012



N =13

AMIN	C8	L8	F8	C9	C10	L10	F10	C11	C12	L12	F12	C13
40.0	1.3340	.7386		.8398	1.3868	.4883		.5405	1.2824	.1684		-.4331
45.0	1.1703	.8420		.9069	1.1924	.5679		.5801	1.0716	.2015		-.3660
50.0	1.0424	.9453		.9798	1.0435	.6490		.6237	.9132	.2364		-.3084
55.0	.9392	1.0492		1.0575	.9259	.7314		.6707	.7907	.2731		-.2578
60.0	.8538	1.1540		1.1396	.8308	.8151		.7208	.6937	.3113		-.2126
65.0	.7818	1.2605		1.2256	.7522	.9003		.7738	.6154	.3509		-.1715
70.0	.7199	1.3688		1.3154	.6861	.9871		.8294	.5510	.3919		-.1336
75.0	.6661	1.4793	1.007344	1.4088	.6296	1.0756	1.215090	.8878	.4972	.4343	2.151818	-.0984
80.0	.6188	1.5925		1.5058	.5808	1.1660		.9487	.4518	.4779		-.0651
85.0	.5767	1.7085		1.6063	.5381	1.2585		1.0122	.4129	.5230		-.0335
90.0	.5391	1.8277		1.7106	.5004	1.3533		1.0782	.3792	.5694		-.0032
95.0	.5052	1.9504		1.8186	.4669	1.4505		1.1470	.3499	.6171		.0259



N =14

AMIN	C8	L8	F8	C9	C10	L10	F10	C11	C12	L12	F12	C13	L14
40.0	1.3492	.7317		.8331	1.1246	.6342		.6002	.6648	.4120		.1214	.1874
45.0	1.1945	.8265		.9076	1.0029	.7112		.6616	.5992	.4571		.1658	.1936
50.0	1.0713	.9215		.9850	.9035	.7894		.7237	.5440	.5035		.2084	.2000
55.0	.9705	1.0172		1.0651	.8207	.8691		.7868	.4969	.5512		.2499	.2068
60.0	.8862	1.1139		1.1480	.7506	.9503		.8511	.4563	.6003		.2905	.2139
65.0	.8145	1.2121		1.2335	.6904	1.0331		.9169	.4210	.6507		.3305	.2213
70.0	.7525	1.3119		1.3217	.6382	1.1175		.9842	.3899	.7024		.3702	.2291
75.0	.6983	1.4136	1.006408	1.4126	.5925	1.2039	1.183993	1.0532	.3625	.7555	1.910553	.4098	.2373
80.0	.6505	1.5176		1.5062	.5520	1.2922		1.1241	.3381	.8102		.4495	.2459
85.0	.6079	1.6239		1.6027	.5159	1.3826		1.1969	.3162	.8663		.4893	.2548
90.0	.5697	1.7328		1.7022	.4835	1.4753		1.2718	.2964	.9240		.5296	.2642
95.0	.5352	1.8446		1.8048	.4542	1.5705		1.3490	.2785	.9834		.5703	.2740

```
// OPTION LINK,NOLISTX,NODUMP
// EXEC FFORTRAN
C       ****************************
C       *                          *
C       * ZDUNEK TRANSFORMATION    *
C       * LOWPASS TO LOWPASS       *
C       * AUTHOR F.W.SHEPHERD      *
C       *                          *
C       ****************************
C
        REAL K
C
 1000 FORMAT(A4,2F11.1)
 1001 FORMAT(//,17X,'S-PLANE',15X,'ST-PLANE',/,13X,'REAL',8X,'IMAG.',
     1       7X,'REAL',8X,'IMAG.',/)
 1002 FORMAT('1OMEGA(Z) =',F12.8,4X,'OMEGA(I) =',F12.8,4X,'K =',F12.8)
 1003 FORMAT(3X,A4,3X,4F12.8)
 1004 FORMAT(34X,2F12.8)
 1005 FORMAT(/)
C
      DATA LBL1/'SKIP'/
      DATA LBL2/'END '/
      READ (1,1000)LABEL,OZ,OI
      K=SQRT((1.+OI)/(1.+OZ))
      WRITE(3,1002)OZ,OI,K
      WRITE(3,1001)
C
   20 READ(1,1000)LABEL,SR,SI
      IF(LABEL-LBL1)21,22,21
   21 IF(LABEL-LBL2)23,60,23
   22 WRITE(3,1005)
      GO TO 20
   23 CONTINUE
      XR=SR
      IF(SI-1000.0)30,25,25
   25 SIN=0.0
      SR=K
      WRITE(3,1004)SIN,SR
      GO TO 20
   30 SIN=SI-OZ
      SID=SI-OI
      IF(SR)40,35,40
   35 SR=SQRT(SIN/SID)
      SIN=0.0
      GO TO 45
   40 CALL CDIV(SR,SIN,SR,SID)
      CALL CROOT(SR,SIN)
      SIN=-ABS(SIN)
   45 SIN=K*SIN
      SR=K*SR
      WRITE(3,1003)LABEL,XR,SI,SIN,SR
      GO TO 20
   60 CALL EXIT
      END
```

```
C      COMPLEX DIVIDE
C
       SUBROUTINE CDIV(A,B,C,D)
       X=1.0/(C*C+D*C)
       Y=X*(A*C+B*D)
       B=X*(B*C-A*C)
       A=Y
       RETURN
       END
C
C      SUBROUTINE CRCOT(X,Y)
C
       SUBRCUTINE CROOT(X,Y)
       R=SQRT(X*X+Y*Y)
       IF(Y)10,20,20
    10 Y=SQRT(0.5*(R-X))
       X=-SQRT(0.5*(R+X))
       GO TO 30
    20 Y=SQRT(0.5*(R-X))
       X=SQRT(0.5*(R+X))
    30 RETURN
       END
/*
// EXEC LNKEDT
// EXEC
```

```
THE FOLLOWING INPUT DATA ARE TAKEN FROM  (CE-1)
THE RESULTS ARE SHOWN IN FIG.V.13

      0.958140   1.96552102
F(S)             -.95814
                 -.619191
       0.0
                 0.619191
                 0.95814
SKIP
P(S)             3.04764878
                 999999.
                 -3.0476488
                 -1.969521
                 -1.88708
SKIP
E(S)-0.13550108-1.0730606
    -0.42873273-.72958889
    -0.60042398
    -0.428732730.72958889
    -0.135501081.0730606
END
/*
```

Grundgrad $n = 4$

δ	τ_0	η (%)	$-\alpha_{1,2}$	$\pm\beta_{1(2)}$	$-\alpha_{3,4}$	$\pm\beta_{3(4)}$
0,01	5,318	42,3	0,853679	0,397227	0,656397	1,171164
0,02	5,739	45,7	0,756312	0,378437	0,588183	1,110755
0,03	6,000	47,7	0,701666	0,368359	0,549937	1,078342
0,04	6,193	49,3	0,663868	0,361589	0,523482	1,056599
0,05	6,346	50,5	0,635087	0,356544	0,503328	1,040431
0,06	6,474	51,5	0,611910	0,352550	0,487088	1,027662
0,07	6,585	52,4	0,592547	0,349261	0,473509	1,017172
0,08	6,682	53,2	0,575945	0,346473	0,461855	1,008307
0,09	6,763	53,9	0,561430	0,344060	0,451656	1,000657
0,1	6,847	54,5	0,548547	0,341938	0,442596	0,993948
0,2	7,387	58,8	0,466127	0,328763	0,384327	0,952858
0,3	7,723	61,5	0,419747	0,321626	0,351183	0,931197
0,4	7,971	63,4	0,387648	0,316787	0,328012	0,916855
0,5	8,169	65,0	0,363213	0,313153	0,310207	0,906306

Grundgrad $n = 5$

δ	τ_0	η (%)	$-\alpha_1$	$-\alpha_{2,3}$	$\pm\beta_{2(3)}$	$-\alpha_{4,5}$	$\pm\beta_{4(5)}$
0,01	7,528	47,9	0,691440	0,652828	0,586607	0,488504	1,147715
0,02	8,015	51,0	0,616598	0,584569	0,565446	0,443006	1,102108
0,03	8,314	52,9	0,574224	0,545848	0,553915	0,417191	1,077333
0,04	8,532	54,3	0,544769	0,518887	0,546086	0,399200	1,060576
0,05	8,706	55,4	0,522263	0,498257	0,540204	0,385419	1,048037
0,06	8,851	56,3	0,504093	0,481580	0,535516	0,374265	1,038084
0,07	8,975	57,1	0,488883	0,467603	0,531633	0,364905	1,029872
0,08	9,084	57,8	0,475819	0,455586	0,528325	0,356847	1,022906
0,09	9,181	58,4	0,464382	0,445054	0,525451	0,349776	1,016874
0,1	9,269	59,0	0,454219	0,435686	0,522912	0,343478	1,011568
0,2	9,869	62,8	0,388956	0,375293	0,506926	0,302630	0,978706
0,3	10,239	65,2	0,352062	0,340935	0,498087	0,279114	0,961086
0,4	10,510	66,9	0,326465	0,316986	0,492016	0,262546	0,949283
0,5	10,727	68,3	0,306946	0,298655	0,487411	0,249742	0,940522

Grundgrad $n = 6$

δ	τ_0	η (%)	$-\alpha_{1,2}$	$\pm\beta_{1(2)}$	$-\alpha_{3,4}$	$\pm\beta_{3(4)}$	$-\alpha_{5,6}$	$\pm\beta_{5(6)}$
0,01	9,882	52,4	0,568554	0,234444	0,523887	0,693616	0,386097	1,128599
0,02	10,422	55,3	0,510119	0,227812	0,472893	0,673496	0,353124	1,092474
0,03	10,752	57,0	0,476837	0,224125	0,443742	0,662406	0,334257	1,072671
0,04	10,993	58,3	0,453616	0,221588	0,423346	0,654819	0,321038	1,059198
0,05	11,183	59,3	0,435828	0,219660	0,407683	0,649087	0,310873	1,049071
0,06	11,341	60,2	0,421437	0,218111	0,394984	0,644498	0,302619	1,041003
0,07	11,477	60,9	0,409370	0,216816	0,384316	0,640681	0,295675	1,034325
0,08	11,595	61,5	0,398991	0,215707	0,375125	0,637420	0,289684	1,028646
0,09	11,701	62,1	0,389894	0,214737	0,367054	0,634577	0,284416	1,023716
0,1	11,797	62,6	0,381801	0,213875	0,359865	0,632061	0,279717	1,019370
0,2	12,446	66,0	0,329625	0,208343	0,313240	0,616067	0,249046	0,992240
0,3	12,843	68,1	0,299969	0,205198	0,286490	0,607111	0,231236	0,977522
0,4	13,134	69,7	0,279320	0,202999	0,267741	0,600910	0,218622	0,967583
0,5	13,365	70,9	0,263532	0,201310	0,253329	0,596178	0,208833	0,960159

Grundgrad $n = 7$

δ	τ_0	η(%)	$-\alpha_1$	$-\alpha_{2,3}$	$\pm\beta_{2(3)}$	$-\alpha_{4,5}$	$\pm\beta_{4(5)}$	$-\alpha_{6,7}$	$\pm\beta_{6(7)}$
0,01	12,341	56,1	0,490144	0,478738	0,386733	0,435468	0,760875	0,317813	1,113506
0,02	12,927	58,8	0,441209	0,431829	0,377574	0,395554	0,742475	0,292533	1,083849
0,03	13,283	60.4	0,413255	0,404981	0,372428	0,372602	0,732246	0,277977	1,067480
0,04	13,542	61,6	0,393719	0,386193	0,368861	0,356482	0,725210	0,267737	1,056292
0,05	13,746	62,5	0,378737	0,371767	0,366138	0,344068	0,719872	0,259839	1,047855
0,06	13,916	63,3	0,366605	0,360076	0,363940	0,333981	0,715584	0,253412	1,041115
0,07	14,061	63,9	0,356425	0,350259	0,362098	0,325491	0,712009	0,247993	1,035522
0,08	14,188	64,5	0,347665	0,341804	0,360514	0,318164	0,708947	0,243311	1,030756
0,09	14,301	65,0	0,339982	0,334384	0,359126	0,311721	0,706273	0,239187	1,026613
0,1	14,403	65,5	0,333144	0,327777	0,357891	0,305975	0,703901	0,235504	1,022953
0,2	15,093	68,6	0,288998	0,285025	0,349900	0,268533	0,688730	0,211353	0,999975
0,3	15,514	70,5	0,263856	0,260599	0,345310	0,246909	0,680161	0,197240	0,987399
0,4	15,821	71,9	0,246329	0,243535	0,342083	0,231686	0,674195	0,187202	0,978858
0,5	16,064	73,0	0,232916	0,230454	0,339593	0.219946	0,669625	0,179390	0,972448

Appendix F

A Numerical Procedure to Determine the Element Values of Brune Sections

Example: Design a lowpass with reflection zeroes at $s_{o1} = \pm j$, $s_{o2} = \pm 2j$ and attenuation poles at $s_{\infty 1} = \pm 3j$, $s_{\infty 2} = \pm 4j$ (all normalized). At $s = 0$, a transducer loss $A_o = 0.2$ dB is postulated.

Solution:

(a) Characteristic function

$$K(s) = \frac{(s^2+1^2)(s^2+2^2)}{(s^2+3^2)(s^2+4^2)} = C\,\frac{s^4+5s^2+4}{s^4+25s^2+144}$$

$$A_o = 10\log_{10}[1+K^2(0)] = 0.2\text{ dB} \rightarrow C = 7.81527$$

(b) Hurwitz polynomial

(II.14): $E(s) = 1.00815s^4 + 3.11292s^3 + 10.17151s^2 + 15.02263s + 18.85463$

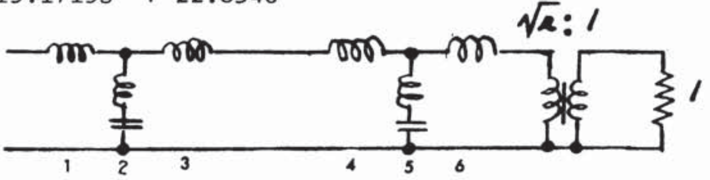
(c) Input impedance and input reactances

$$z_{in} = \frac{E(s)-F(s)}{E(s)+F(s)} = \frac{0.00815s^4 + 3.1129s^3 + 5.1715s^2 + 15.0226s + 14.8546}{2.00814s^4 + 3.1129s^3 + 15.1715s^2 + 15.0226s + 22.8546}$$

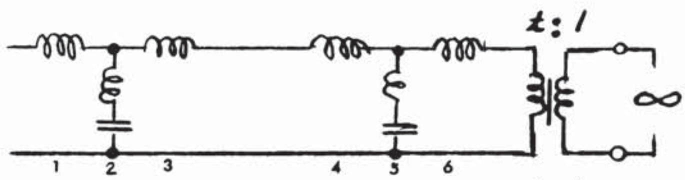
$$z_{oc} = \frac{E_e - F_e}{E_o + F_o} = \frac{0.00815s^4 + 5.1715s^2 + 14.8546}{3.1129s^3 + 15.0226s}$$

$$z_{sc} = \frac{E_o - F_o}{E_e + F_e} = \frac{3.1129s^3 + 15.1226s}{2.00815s^4 + 15.1715s^2 + 22.8546}$$

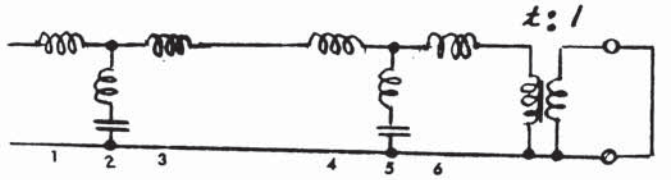
$$z_{in} = \frac{E(s)-F(s)}{E(s)+F(s)} \rightarrow$$



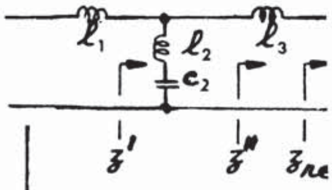
$$z_{oc} = z_{11}(s) = \frac{E_e - F_e}{E_o + F_o} \rightarrow$$



$$z_{sc} = \frac{1}{y_{11}(s)} = \frac{E_o - F_o}{E_e + F_e} \rightarrow$$



$$z_{11} = \frac{n(s^2)}{s\,d(s^2)} = \frac{(n_8 s^8 + n_6 s^6 + n_4 s^4 + n_2 s^2 + n_0)}{s\,(d_6 s^6 + d_4 s^4 + d_2 s^2 + d_0)} \longrightarrow$$



$$s\,\ell_1 \Big|_{j\omega} = z(s)\Big|_{j\omega} \longrightarrow \ell_1 = \frac{n(s^2)}{s\,d(s^2)}\Bigg|_{s^2=-\omega}$$

Horner's Method for $s^2 = -\omega^2 = -9$

$n_8 = 0$	$n_6 = 0$	0.0081529	5.1715	14.8546	$n(s^2)$
		0	-0.075376	-45.8832	
		0.0081529	5.098140	$\boxed{-31.0286}$	$\longrightarrow n(-9)$

$d_6 = 0$	$d_4 = 0$	3.11292	15.02263	$d(s^2)$
		0	-28.01628	
		3.11292	$\boxed{-12.99365}$	$\longrightarrow d(-9)$

$\ell_1 = -0.2$

$$z' = z - s\,\ell_1 = \frac{n(s^2) - s^2 \ell_1 d(s^2)}{s\,d(s^2)} = \frac{n'(s^2)}{s\,d(s^2)} = \frac{(s^2+\omega^2)(n_6'' s^6 + n_4'' s^4 + n'' s + n_0'')}{s\,d(s^2)}$$

$n' = 0$	$n' = 0$	0.834109	9.157495	14.854627	$n'(s)$
		0	-7.506981	-14.854628	
$n'' = 0$	$n'' = 0$	0.834109	1.650514	0	$n''(s)$
		0	-7.50698		
		0.834109	-5.85646		$\longrightarrow n''(-9)$

$$y' = \frac{s\,d(s^2)}{(s+\omega^2)\,n''(s^2)} = \frac{A\,s}{(s^2+\omega^2)} + \frac{s\,d(s^2)}{n''(s^2)} = \frac{A\,s}{(s^2+\omega^2)} + y''$$

$$A = \frac{d(s^2)}{n''(s^2)}\Bigg|_{s^2=-9} ; \quad \ell_2 = \frac{1}{A} ; \quad c_2 = \frac{A}{\omega}$$

$A = 2.21868$

$\ell_2 = 0.45071$

$c_2 = 0.2465$

Comparison of coefficients :

$$\left.\begin{aligned} d_0 &= A\,n_0'' + \omega\,d_0' \\ d_2 &= A\,n'' + \omega\,d_2' + d_0' \\ d_4 &= A\,n_4'' + \omega\,d_4' + d_2' \\ d_6 &= A\,n_6'' + + d_4' \end{aligned}\right\}$$

$d_0' = [\,d_0 - A\,n_0''\,]/\omega$

$d_2' = [\,d_2 - A\,n'' - d_0'\,]/\omega$

$d_4' = [\,d_4 - A\,n_4'' - d_2'\,]/\omega$

$d_0' = 1.26229$

$d_2' = 0$

$d_4' = 0$

$$z'' = \frac{(n_6'' s^6 + n_4'' s^4 + n_2'' s^2 + n_0'')}{s\,(d_6' s^6 + d_4' s^4 + d_2' s^2 + d_0')} ; \quad \ell_3 = -\frac{\ell_1 \ell_2}{\ell_1 + \ell_2}$$

$\ell_3 = 0.64509$

$$z_{red} = z'' - s\,\ell_3 = \frac{(n_6''' s^6 + n_4''' s^4 + n_2''' s^2 + n_0''')}{s\,d(s^2)}$$

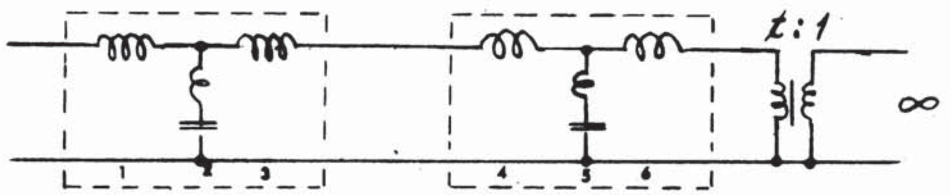
$$= \frac{0.0198 s^2 + 1.6505}{1.2622\,s}$$

The realization is carried out by the decomposition of z_{oc} where

$$z_{oc} = \frac{n(s)}{s\,d(s)} = \frac{n_4 s^4 + n_2 s^2 + n_o}{s(d_2 s^2 + d_o)} = \frac{\text{4th degree}}{\text{3rd degree}}$$

For the numerical calculations, the form shown in Fig. F.1 is quite useful. As prepared, this calculation form can be used for z_{oc} = (8th degree)/(7th degree) reactances. The z_{oc} above is a special case where $n_8 = n_6 = d_6 = d_4 = 0$. The numerical values for the others are inserted in their proper locations and the calculations are carried out with these numbers. The same form can be used again for the reduced impedance z_{red}(2nd degree)/(1st degree) although, manually, one would use a simpler method.

Based on these calculations the following circuit results:



$l_1 = -0.265$ $l_2 = 0.450$ $l_3 = 0.645$ $l_4 = -0.066$ $l_5 = 0.0817$ $l_6 = 0.343$

$c_2 = 0.246$ $c_5 = 0.765$

$\omega_2 = 3.0$ $\omega_5 = 4.0$

(e) <u>The turns ratio of the ideal transformer</u>

Neither from z_{oc} nor from z_{sc}, the turns ratio t of the ideal transformer can be calculated. Two methods are conventionally used to calculate t. The first is easier but applies only if there is no attenuation pole at the origin. In this case, the transmission network is a through connection at s = 0. Then from,

z_{in}

$$z_{in}(0) = \frac{e_o - f_o}{e_o + f_o} = t^2 \cdot 1 \rightarrow t = \sqrt{\frac{e_o - f_o}{e_o + f_o}}$$

which yields in the numerical example

$$t^2 = \frac{14.8546}{22.8546} \rightarrow t = 0.806$$

This method fails if there is an attenuation pole at s = 0

By the second method, one establishes first a suitable output reactance, for instance

$$z_{sc,2} = \frac{1}{y_{22}} = \frac{E_o - F_o}{E_e - F_e} = \frac{3.1129s^3 + 15.1226s}{0.00815s^4 + 5.1715s^2 + 14.8546}$$

From $z_{sc,2}$, the numerical value of the rightmost element is

$$\lim \frac{1}{s} z_{sc,2}(s) \Big|_{s = \pm 4j} = \frac{\ell_6}{t^2} = 0.527$$

from which one obtains again

$$t^2 = \frac{\ell_6}{0.527} = 0.651 \rightarrow t = 0.806$$

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